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PRIVATIZATION AND GOVERNMENT PREFERENCE IN A BERTRAND MODEL

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Abstract: We will consider a mixed Bertrand duopoly model (that means, two firms decide simultaneously their prices for a substitutable good) to study the relationship between the privatization of a state-owned public firm and government preferences for tax revenue. In the model, we assume that the government imposes a specific tax rate on the quantity produced by each firm. Furthermore, the public firm aims to maximize social welfare, whereas the government's objective function is a weighted sum between social welfare and tax revenue. Of course, the private firm aims to maximize its own profit. We also present comparative static results.

keywords: Modeling, Optimization, Industrial Organization, Game Theory.

1. INTRODUCTION

Competition between public and private firms exists in a range of industries like telecommunications, electricity, natural gas, airlines industries, and services including hospitals, banking and education. Some authors studied mixed oligopolies under quantity competition and some others considered price competition. Ogawa and Kato [5] consider a Bertrand model in which firms produce homogenous goods and have quadratic cost functions. They compare the prices set by firms when prices are set simultaneously and sequentially. White [6] and Fjell and Heywood [2] introduced a subsidy into the mixed model. In these studies, the objective function of both the government and the public firm is the social welfare. Matsumura [4] considered an objective func-

tion that is a weighted average of a modified social welfare and the profit of the firm. The modified social welfare allows the government to prefer consumer surplus to the profits of the two firms.

Kato [3] uses a mixed Cournot model to study the relationship between privatization and government preferences for tax revenue. He assumes that the government prefers tax revenue to the social welfare, whereas the public firm only cares about the social welfare. Ferreira and Ferreira [1] did a similar analysis in a Stackelberg duopoly model with the public firm as the leader.

In this paper, we also study the relationship between privatization and government preferences for tax revenue, but by considering a Bertrand model, instead of a Cournot model. Furthermore, we consider that firms produce differentiated goods, instead of homogenous goods. We get that if the government puts a sufficiently larger weight on tax revenue than on the sum of both surpluses, it will not privatize the public firm. In contrast, if the government puts a moderately larger weight on tax revenue than on the sum of both surpluses, it will privatize the public firm.

2. THE MIXED DUOPOLY

We consider a mixed duopoly with firms F_1 and F_2 being, respectively, the public and the private firms. Both firms produce substitutable goods. The demand functions are given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2}, \quad i \neq j, \quad i, j = 1, 2,$$

where p_i is the market price of the good produced by firm F_i , q_i is the quantity produced by firm F_i , $b \in (0, 1)$ measures the degree to which goods are substitutes and $a > 0$. For simplicity, we assume $a = 1$ and $b = 0.5$. So,

$$q_i = \frac{2}{3}(1 - 2p_i + p_j), \quad i \neq j, \quad i, j = 1, 2.$$

The decision variables of the firms are the prices for the goods they produce, and they choose their values simultaneously. So, both firms act as Bertrand players in the market. The marginal cost of production of both firms is c , that, without loss of generality, we assume equal to zero. Furthermore, we assume that the government imposes a specific tax rate on both firms. So, the profit function π_i of firm F_i is given by

$$\pi_i = (p_i - t)q_i, \quad i \neq j, \quad i, j = 1, 2,$$

where t is the specific tax rate.

Social welfare W , which is the sum of consumer surplus (CS) and producer surplus, is given by

$$W = CS + \pi_1 + \pi_2,$$

where

$$CS = \frac{2}{3}(p_1^2 - p_1 p_2 + p_2^2 + 1 - p_1 - p_2).$$

The public firm F_1 aims to maximize the social welfare W , and the private firm F_2 aims to maximize its profit π_2 . The government's payoff is given by

$$U = W + (1 + \alpha)T,$$

where $T = t(q_1 + q_2)$ is the tax revenue, and α is the parameter representing the weight of the government preference for the tax revenue. Since we are interested in the case where the government puts a larger weight on T than on W , we consider $\alpha \geq 0$. Furthermore, we are also interested just in inner solutions; so, we restrict the values of α to $\alpha > 1/22$.

The model is a two-stage game. In the first stage, the government chooses the specific tax rate t . In the second stage, both firms decide the prices p_1 and p_2 , simultaneously. To obtain a subgame perfect equilibrium, the game is solved by backwards induction.

In the mixed Bertrand duopoly model presented above, we get the following results ¹.

Proposition 1. In the Bertrand mixed duopoly, as described above, the tax rate and prices, at equilibrium, are given by

$$t^M = \frac{77\alpha - 15}{2(77\alpha + 31)},$$

$$p_1^M = \frac{2(22\alpha - 1)}{77\alpha + 31},$$

$$p_2^M = \frac{99\alpha + 7}{77\alpha + 31}.$$

We observe that for $\alpha > 15/77$, the optimal tax rate is positive; for $1/22 < \alpha < 15/77$, the optimal tax rate is negative. From the expressions above, we obtain the following result:

Proposition 2. At equilibrium, the aggregate quantity, the social welfare and the government's payoff function are given by

$$Q^M = \frac{121(\alpha + 1)}{3(77\alpha + 31)},$$

$$W^M = \frac{2783(\alpha + 1)^2}{3(77\alpha + 31)^2},$$

$$U^M = \frac{121(\alpha + 1)^2}{6(77\alpha + 31)}.$$

3. THE PRIVATIZED DUOPOLY

Now, suppose that the public firm is privatized. So, both firms aim to maximize their own profits.

In the privatized Bertrand duopoly model presented above, we get the following results ².

Proposition 3. In the Bertrand privatized duopoly, as described above, the tax rate and prices, at equilibrium, are given by

$$t^P = \frac{3\alpha - 1}{2(3\alpha + 1)},$$

$$p_1^P = p_2^P = \frac{2\alpha}{3\alpha + 1}.$$

We observe that for $\alpha > 1/3$, the optimal tax rate is positive; for $1/22 < \alpha < 1/3$, the optimal tax rate is negative. From the expressions above, we obtain the following result:

Proposition 4. At equilibrium, the aggregate quantity, the social welfare and the government's payoff function are given by

$$Q^P = \frac{4(\alpha + 1)}{3(3\alpha + 1)},$$

$$W^P = \frac{4(\alpha + 1)^2}{3(3\alpha + 1)^2},$$

$$U^P = \frac{2(\alpha + 1)^2}{3(3\alpha + 1)}.$$

¹Throughout the paper, we use the notation superscript M to refer to the mixed competition.

²Throughout the paper, we use the notation superscript P to refer to the privatized competition.

4. EFFECTS OF PRIVATIZATION

In this section, we compare the mixed and privatized duopoly equilibria.

$$\begin{aligned} t^M - t^P &= \frac{8(\alpha + 1)}{(3\alpha + 1)(77\alpha + 31)}, \\ Q^M - Q^P &= \frac{(\alpha + 1)(55\alpha - 3)}{3(3\alpha + 1)(77\alpha + 31)}, \\ W^M - W^P &= \frac{(\alpha + 1)^2(1331\alpha^2 - 2398\alpha - 1061)}{3(3\alpha + 1)^2(77\alpha + 31)^2}, \\ U^M - U^P &= \frac{(\alpha + 1)^2(55\alpha - 3)}{6(3\alpha + 1)(77\alpha + 31)}. \end{aligned}$$

These results lead to the following. Privatization will decrease the optimal tax rate imposed by the government. If $1/22 < \alpha < 3/55$, privatization will increase the aggregate quantity in the market; and if $\alpha > 3/55$, privatization will decrease the aggregate quantity in the market. If $1/22 < \alpha < (109 + 32\sqrt{23})/121$, privatization will increase the social welfare; and if $\alpha > (109 + 32\sqrt{23})/121$, privatization will decrease the social welfare. If $1/22 < \alpha < 3/55$, privatization will increase the government's payoff; and if $\alpha > 3/55$, privatization will decrease the government's payoff.

5. CONCLUSION

In this paper, we analysed the relationship between the privatization of a public firm and government preferences for tax revenue in a differentiated Bertrand duopoly model. We concluded that if the government puts a sufficiently larger weight on tax revenue than on the sum of both surpluses, it will not privatize the public firm. In contrast, if the government puts a moderately larger weight on tax revenue than on the sum of both surpluses, it will privatize the public firm.

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References

- [1] F.A. Ferreira, and F. Ferreira, "Privatization and government preferences in a mixed duopoly: Stackelberg versus Cournot," In Tenreiro Machado et al. (Eds.) *Discontinuity and Complexity in Nonlinear Physical Systems*. DOI: 10.1007/978-3-319-01411-1-24, Springer International Publishing Switzerland, pp. 421-430, 2014.
- [2] K. Fjell, and J. Heywood, "Mixed oligopoly, subsidization and the order of firms's moves: the relevance of privatization," *Economics Letters*, Vol. 53, pp. 411-416, 2004.
- [3] H. Kato, "Privatization and government preference," *Economics Bulletin*, Vol. 12, No. 40, pp. 1-7, 2008.
- [4] T. Matsumura, "Partial privatization in mixed Duopoly," *Journal of Public Economics*, Vol. 70, No. 40, pp. 473-483, 1998.
- [5] A. Ogawa, and K. Kato, "Price competition in a mixed duopoly," *Economics Letters*, Vol. 12, pp. 4-15, 2006.
- [6] M. White, "Mixed oligopoly, privatization and subsidization," *Economics Letters*, Vol. 53, pp. 189-195, 1996.