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PRICE-SETTING MIXED TRIOPOLIES

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Abstract: We will consider two models describing certain market structures: (i) a domestic market in which a public firm (whose objective is to maximize social welfare) competes with two private firms (whose objective is to maximize their own profits); and (ii) an international market in which a domestic public firm competes with one domestic private firm and one foreign private firm. In both situations, firms decide simultaneously the price for their substitutable goods. The main purpose of the paper is to present and to compare the equilibrium outcomes of the two triopoly models.

keywords: Modeling, Optimization, Industrial Organization, Game Theory.

1. INTRODUCTION

There are several research works on mixed oligopolies, in which public firms compete with private firms (see, for example, Ferreira and Ferreira [1], Fjell and Pal [2], Matsumura [3], Ogawa and Kato [4], Ohnishi [5] and White [6]). Public firms maximize social welfare, whereas private firms maximize their own profits. We can find public firms competing with private firms in many industries such as airlines, automobiles and banking. These mixed competitions can also be found in international markets. Some literature examines mixed models with quantity competition and some other examines price quantity. Some authors study mixed models where the decisions are taken simultaneously and some others consider mixed models with sequential decisions. Furthermore, there are papers presenting models with homogeneous goods and papers presenting models with differentiated

goods.

In this paper, we start from Ohnishi [5], who examines domestic (respectively, international) mixed competition with a public firm competing on price with a domestic (respectively, foreign) private firm. We do a similar analysis in a framework of a market with three firms. We consider a mixed triopoly with one public firm F_0 and two private firms F_1 and F_2 . In the international competition, the private firm F_2 is a foreign firm. Furthermore, and also in contrast with Ohnishi [5], we assume that the home government imposes a tariff on the imported good. We compute the equilibrium outcomes of the domestic and international mixed competitions and we also compare the results obtained in both models. In contrast with the results obtained by Ohnishi [5], in our triopoly model, the domestic private firm's profit in the domestic competition is higher than the foreign private firm's profit in the international mixed competition.

The remainder of this paper is organized as follows. In section 2, we present the domestic mixed triopoly model and the corresponding equilibrium outcomes. In section 3, we present the international mixed triopoly model and the its equilibrium outcomes. Section 4 compares the results of the two models. Section 5 concludes the paper.

2. DOMESTIC MIXED TRIOPOLY

We consider a mixed triopoly with one public firm F_0 and two private firms F_1 and F_2 . Assume that firms produce differentiated goods and the market demand is given by

$$p_i = \alpha - q_i - \beta Q_{-i},$$

where p_i is the price of good i , q_i is the output of good i , α is a positive constant and $\beta \in (0, 1)$ is a constant that captures the extent of good differentiation, $Q_{-i} = \sum_{j \neq i} q_j$ is the total output of all firms other than F_i . Thus, we get

$$q_i = \frac{\alpha(1 - \beta) - (1 + \beta)p_i + \beta(p_j + p_k)}{1 + \beta - 2\beta^2},$$

with $i, j, k = 0, 1, 2$ and $i \neq j \neq k$. For simplicity, we assume $\alpha = 1$ and $\beta = 0.5$. The consumer surplus is, therefore, given by

$$CS = \frac{3(p_0^2 + p_1^2 + p_2^2 + 1)}{4} - \frac{p_0p_1 + p_0p_2 + p_1p_2 + p_0 + p_1 + p_2}{2}.$$

Each firm's profit function is defined by

$$\pi_i = (p_i - c)q_i, i = 0, 1, 2,$$

where $c > 0$ is the unit production cost. We assume $c < 1$ to assure that production outputs are positive. Social welfare is defined as the sum of consumer and producer surplus:

$$W = CS + \pi_0 + \pi_1 + \pi_2.$$

The decision variables of the firms are the prices for the goods they produce, and they choose their values simultaneously.

The public firm F_0 aims to maximize the social welfare W , and the private firms F_1 and F_2 aim to maximize their own profits π_1 and π_2 , respectively.

In the domestic price-setting triopoly model presented above, we get the following results ¹.

Proposition 1. In the domestic price-setting triopoly, as described above, the equilibria outcomes are given by

$$\begin{aligned} p_0^D &= \frac{11c + 2}{13}, \\ p_1^D &= p_2^D = \frac{10c + 3}{13}, \\ q_0^D &= \frac{1 - c}{2}, \\ q_1^D &= q_2^D = \frac{9(1 - c)}{26}, \\ \pi_0^D &= \frac{(1 - c)^2}{13}, \\ \pi_1^D &= \pi_2^D = \frac{27(1 - c)^2}{338}, \\ CS^D &= \frac{323(1 - c)^2}{676}, \\ W^D &= \frac{483(1 - c)^2}{676}. \end{aligned}$$

¹Throughout the paper, we use the notation superscript D to refer to the domestic competition.

3. INTERNATIONAL MIXED TRIOPOLY

In this section, we consider an international mixed triopoly with one public firm F_0 , one domestic private firm F_1 and one foreign private firm F_2 . The utility, demand, and profit functions are the same as those considered in the previous section. Domestic social welfare is now given by

$$\begin{aligned} W &= CS + \pi_0 + \pi_1 \\ &= \frac{3(1 - p_0^2 - p_1^2 + p_2^2)}{4} \\ &\quad + \frac{p_0p_1 - p_2 + 2c(p_0 + p_1 - p_2 - 1)}{2}. \end{aligned}$$

We also assume that, in this international framework, the home government imposes a tariff t on the imported goods. Therefore, the profit function of the foreign firm is defined by

$$\pi_2 = (p_2 - c - t)q_2.$$

The payoff function of the home government is defined by

$$U = W + tq_2.$$

So, in this case, the game has two stages:

- In the first stage, the home government chooses the imported tariff t ;
- In the second stage, all firms choose, simultaneously, the prices for their goods.

In the international price-setting triopoly model presented above, we get the following results ².

Proposition 2. In the domestic price-setting triopoly, as described above, the equilibria outcomes are given by

$$\begin{aligned} t &= \frac{1295(1 - c)}{6683}, \\ p_0^I &= \frac{6166c + 517}{6683}, \\ p_1^I &= \frac{5132c + 1551}{6683}, \\ p_2^I &= \frac{4577c + 2106}{6683}, \\ q_0^I &= \frac{8789(1 - c)}{13366}, \\ q_1^I &= \frac{4653(1 - c)}{13366}, \\ q_2^I &= \frac{2433(1 - c)}{13366}, \\ \pi_0^I &= \frac{4543913(1 - c)^2}{89324978}, \end{aligned}$$

²Throughout the paper, we use the notation superscript I to refer to the international competition.

$$\begin{aligned}\pi_1^I &= \frac{7216803(1-c)^2}{89324978}, \\ \pi_2^I &= \frac{1973163(1-c)^2}{89324978}, \\ CS^I &= \frac{89208011(1-c)^2}{178649956}, \\ W^I &= \frac{112729443(1-c)^2}{178649956}.\end{aligned}$$

4. COMPARISONS

In this section, we compare the domestic and international mixed triopoly equilibria. The results are stated in the following proposition.

Proposition 3. In the equilibrium outcomes of the domestic and international mixed triopoly models, we have that

$$\begin{aligned}\pi_0^D &> \pi_0^I, \quad \pi_1^D < \pi_1^I, \quad \pi_2^D > \pi_2^I, \\ CS^D &< CS^I, \quad W^D > W^I.\end{aligned}$$

5. CONCLUSION

In this paper, we studied domestic and international mixed triopoly models with price-setting competition. We have seen that the public firm's profit and the social welfare are higher in the domestic competition than in the international competition. Furthermore, the domestic private firm's profit in the domestic competition is also higher than the foreign private firm's profit in the international mixed competition. However, the domestic private firm's profit and the consumer surplus are lower in the domestic competition than in the international competition.

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