

Kinematic Analysis and Modelling of Biped Locomotion Systems

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Abstract

This paper presents the kinematic study of robotic biped locomotion systems. The main purpose is to determine the kinematic characteristics and the system performance during walking. For that objective, the prescribed motion of the biped is completely characterised in terms of five locomotion variables: step length, hip height, maximum hip ripple, maximum foot clearance and link lengths. In this work, we propose four methods to quantitatively measure the performance of the walking robot: energy analysis, perturbation analysis, lowpass frequency response and locomobility measure. These performance measures are discussed and compared in determining the robustness and effectiveness of the resulting locomotion.

Keywords: *Biped Locomotion, Motion Variables, Kinematics, Motion Planning, Performance Measure*

Introduction

In recent years a growing field of research in biped locomotion has resulted in a variety of prototypes whose characteristics, in limited sense, resemble the biological systems (Golden, 1990; Kajita, 1994). This new generation of innovative robotic systems, for real-world applications, is bringing up fascinating perspectives in research and development work. The control of legged vehicles is a difficult and multidisciplinary problem and, even today, is regarded as the most crucial aspect of locomotion (Hemami, 1983). Vukobratovic *et al.* (1990) have proposed models and mechanisms to explain biped locomotion. On the other hand, Raibert and his colleagues built various biped walking machines. Extensive considerations of the major issues with dynamic balance have been presented by Raibert (1986) and Todd (1985). In another perspective, Galhano *et al.* (1992) demonstrated the appeal of implementing biological-inspired schemes in the analysis of mechanical systems. The theories and algorithms guiding this research suggested specific models to apply in biped robots.

In this line of thought, several researchers are in pursuit of better walking robots and continue to refine their models of locomotion (Kun, 1996; Yamaguchi, 1996). The main objective is to design and control an optimum leg geometry that provides good energy efficiency, the required working volume and simplicity in the structure. The movement of the robotic biped will be produced by joint actuators and will be moderated by environmental aspects. Therefore, the resultant motion depends on those factors influencing the structural and functional characteristics of the robot (kinematics problem) and those related to physical phenomena, such as gravity, friction and reaction forces (dynamics problem). This paper concentrates on the kinematic study of a planar biped model with six degrees of freedom and rotational joints. The main purposes are threefold:

- To gain insight into the phenomenon of biped walking.
- To characterise the biped motion in terms of a set of locomotion variables.
- To establish the correlation among these locomotion variables and the system performance.

A wide variety of gait patterns are developed by prescribing the cartesian trajectories of the hip and the lower extremities of the leg. For the analysis of the resulting motion, various kinematic performance measures are proposed and discussed. These kinematic indices can be regarded as quantitative measures of the biped's ability in transporting the section of the body from an initial position to a desired position through the action of the lower limbs. The performance measures and the associated graphical results are used for evaluating the different locomotion patterns.

The remainder of the paper is organised as follows. A short description of the biped model is given in section “Biped Model”. In section “Forward motion planning” we describe the method used to plan the kinematic trajectories of the biped robot. In section “Performance evaluation” various kinematic performance measures are proposed and discussed mathematically. Given this background, several simulation examples are presented in section “Simulation results” to illustrate the application of these methods. Finally, in section “Conclusions” we outline the main conclusions and the perspectives for future research.

Biped Model

Figure 1 shows the planar biped model and the conventions used throughout this paper.

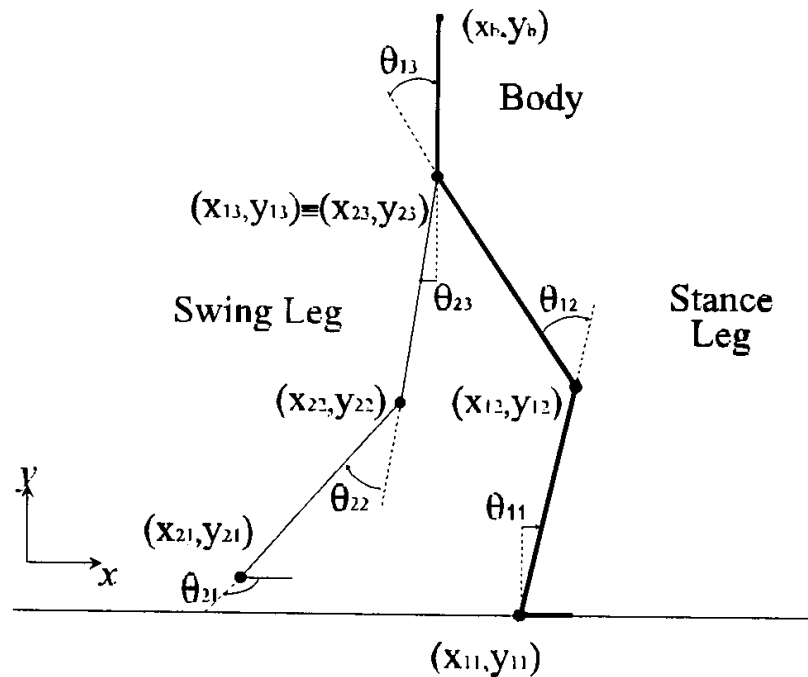


Fig. 1 Planar biped model

The kinematic model consists of seven links in order to approximate locomotion characteristics similar to those of the lower extremities of the human body (*i.e.*, body, thigh, shank and foot). In the present study, we consider a planar biped model with rigid links interconnected through rotational joints. During locomotion, the stance leg (*i.e.*, the limb that is on the ground) has a degree of freedom formed in the contact between the foot and the ground. A step in the gait of a biped consists of two phases:

- Single support phase in which one leg is in contact with the ground and the other leg swings forward.
- Exchange of support in which the legs trade role.

In the single support phase, the stance leg is in contact with the ground and carries the weight of the body, while the swing leg moves forward in preparation for the next step. The impact of the swing leg is assumed to be perfectly inelastic and the friction is sufficient to ensure that no slippage occurs. Additionally, we consider that the support shifts instantaneously from one limb to the other.

Forward Motion Planning

A fundamental problem in robotics is to determine the trajectories that allow the biped robot to walk more skilfully (Shih, 1993). In early work, the determination of the biped trajectories was made largely on the basis of experience and intuition (*e.g.*, recording kinematic data from human locomotion (Silva, 1996)). In this work, the motion planning is accomplished by prescribing the Cartesian trajectories of the body and the lower extremities of the leg. Furthermore, the prescribed motion is completely characterised in terms of five locomotion variables. Simultaneously, we determine the relation between

the locomotion variables and the trajectories physically admissible. In this context, we introduce the concept of locomotion workspace.

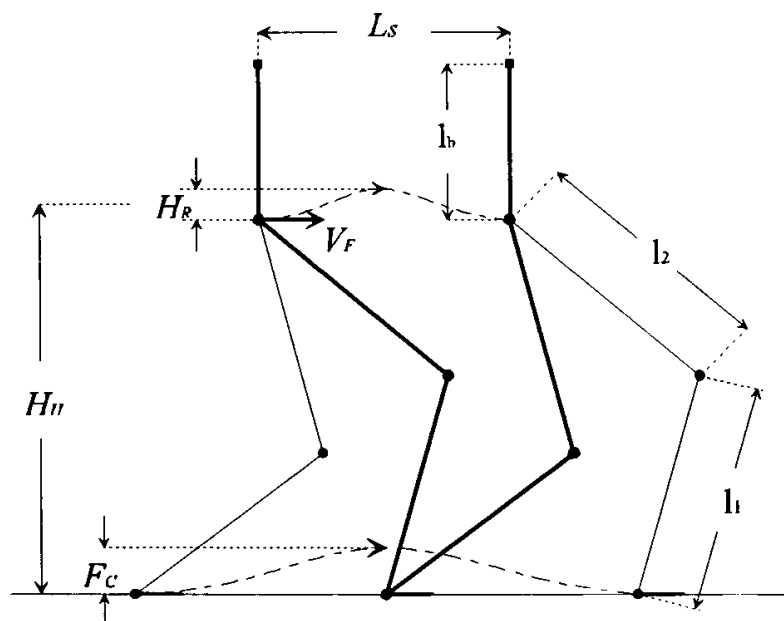


Fig. 2 Locomotion variables

Locomotion variables

The prescribed motion of the biped is completely characterised in terms of the following variables: length of the step L_S , hip height H_H , maximum hip ripple H_R , maximum foot clearance F_C and leg link lengths l_1 and l_2 (see Fig. 2). The hip height is defined as the mean height of the hip along the cycle of walking. The hip ripple is measured as the peak-to-peak oscillation magnitude. During the experiments, we examine the role of the link lengths considering that $l_1 + l_2$ assumes a constant value equal to 1 meter.

Development of the kinematic trajectories

The proposed algorithm accepts the hip and feet's trajectories as inputs and, by means of an inverse kinematics algorithm, generates the related joint trajectories. To improve the smoothness of the motion we impose two additional restrictions: the body maintains an erect posture during locomotion and the forward velocity is constant. Therefore, the horizontal trajectory of the hip is implemented using a constant progression velocity V_F . One trajectory that undergoes smooth motion is the flat trajectory in which the stance leg adjusts itself so that the hip maintains a constant height. It is also of interest to exploit trajectories in which the hip translates with some vertical motion. In order to simplify the problem, we consider that such motions are produced based on sinusoidal functions.

In dynamic walking, at each footfall, the system may suffer impacts and incurs on additional accelerations that influence the forward velocity (Zheng, 1984). For this reason, we must impose a set of conditions (continuity conditions) on the leg velocities so that the feet are placed on the ground while avoiding the impacts. We denote the moment of exchange of support as time t_I , and by t_I^- and t_I^+ the times before and after the impact occurs, respectively. For smooth exchange-of-support, we require the angular velocities, before and after, to be identical, that is:

$$\dot{\theta}_{2i}(t_I^-) = \dot{\theta}_{1i}(t_I^+) \quad 1 \leq i \leq 3 \quad (1)$$

The kinematic relations have been used and the differential problem solved to obtain the cartesian velocities immediately before and after contact. These equations constrain the biped to zero velocity foot falls. The foregoing derivation determined conditions for smooth exchange-of-support. In accordance, the equation of the tip of the swing leg along the x -axis is computed by summing a linear function with a sinusoidal function:

$$x_{2I}(t) = 2V_F \left[t - \frac{1}{2\pi f} \sin(2\pi f t) \right] \quad (2)$$

where f is the step frequency (number of steps per unit of time).

The vertical motion, that allows the foot to be lifted off the ground, is implemented using:

$$y_{2I}(t) = \frac{F_C}{2} [1 - \cos(2\pi f t)] \quad (3)$$

The trajectory generator is responsible for producing a motion that synchronises and coordinates the leg behaviour. In this perspective, we assure that the swing limb arrives at the contact point when the upper body is properly centred with respect to the two lower limbs.

Workspace analysis

One pertinent question involving the motion planning is to determine which trajectories are physically realisable. Upon this point, we define the locomotion workspace of the biped robot as the set of all physically possible hip heights and step lengths, relative to a given reference trajectory (see Fig. 3). The locomotion workspace, as defined above, can be investigated by solving the kinematic equations given two kinds of restrictions:

- Sinusoidal restrictions due the type of velocity and acceleration trajectories used.
- Knee-foot restrictions since the legs are constrained by ground contact and the knee joints lock at certain positions (similar to human knee).

In this way, all the results derived ahead are capable of displaying the performance measure for interpretation over the entire stand-up workspace.

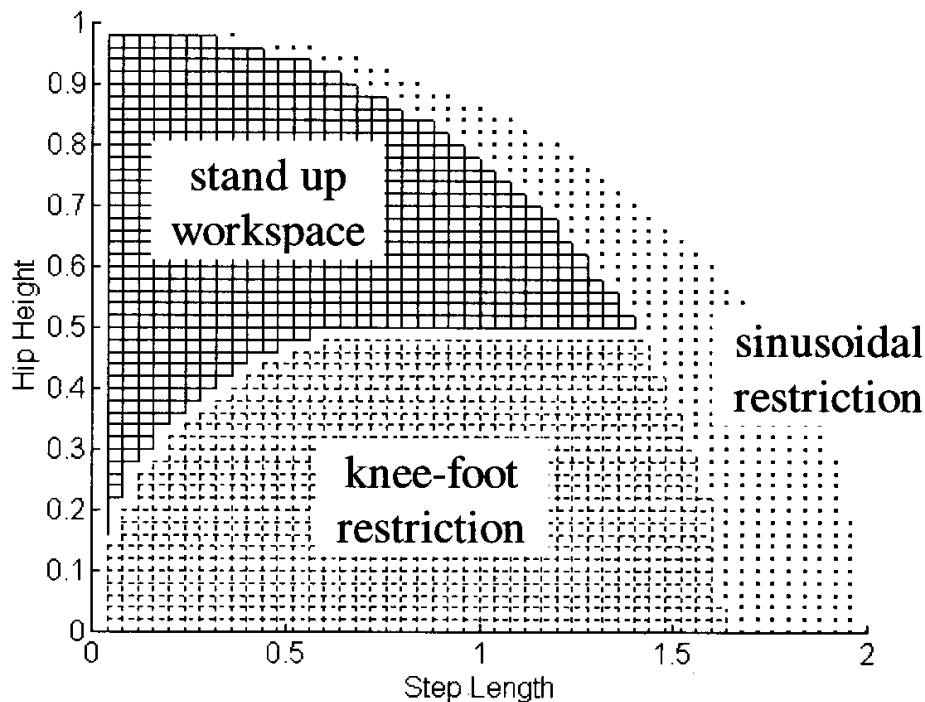


Fig. 3 Workspace boundaries of the biped robot with $l_1 = l_2 = 0.5$

Performance Evaluation

This section covers the implementation of different performance measures used in the evaluation of the biped's kinematic performance. The kinematic analysis is a challenging problem since the biped motion is characterised by gait patterns with complexity of large degrees of freedom (*i.e.*, multidimensional problem). In mathematical terms, we shall provide four global measures of the overall dexterity of the mechanism in some average sense. The aim is to verify whether a correlation between different viewpoints could be found in walking.

Energy analysis

The essence of locomotion is to move smoothly the section of the body from one place to another with some restrictions in terms of execution time. This makes the forward velocity V_F an important variable in determining the walking performance. On the other hand, the objective function should be expressed in the coordinate system on which movement control occurs. This appears to favour the use of joint coordinates to calculate the performance measure.

In this line of thought, we intent to determine the energy characteristics of the rotational-to-linear motion system. This motion conversion is concerned with taking the rotational motion from the joint actuators and producing a linear motion at the body. As a means of inferring the energy flow, we suppose that the translational system exhibits a fictional friction acting on the tip of the body. The linear relationship between the force and the velocity is given by,

$$F(t) = Bv_b(t) \quad (4)$$

where B is the viscous-friction coefficient. For purposes of analysis, suppose that the value of the fictitious friction coefficient is constant $B=1$ Ns/m. The velocity at the body can be obtained, as function of the joint variables, by differentiating the forward kinematic equations:

$$\dot{x}_b = \Gamma_{X_1}(\theta) \cdot \dot{\theta}_{11} + \Gamma_{X_2}(\theta) \cdot \dot{\theta}_{12} + \Gamma_{X_3}(\theta) \cdot \dot{\theta}_{13} \quad (5)$$

$$\dot{y}_b = \Gamma_{Y_1}(\theta) \cdot \dot{\theta}_{11} + \Gamma_{Y_2}(\theta) \cdot \dot{\theta}_{12} + \Gamma_{Y_3}(\theta) \cdot \dot{\theta}_{13} \quad (6)$$

In order to assure an erect posture of the body, the transfer relationships between the joint space and the operational space, along the x and y -axis, are expressed by:

$$\Gamma_{X_1}(\theta) = l_1 \cos \theta_{11} + l_2 \cos(\theta_{11} + \theta_{12}) + l_b \quad (7)$$

$$\Gamma_{X_2}(\theta) = l_2 \cos(\theta_{11} + \theta_{12}) + l_b \quad (8)$$

$$\Gamma_{X_3}(\theta) = l_b \quad (9)$$

and

$$\Gamma_{Y_1}(\theta) = -l_1 \sin \theta_{11} - l_2 \sin(\theta_{11} + \theta_{12}) - l_b \quad (10)$$

$$\Gamma_{Y_2}(\theta) = -l_2 \sin(\theta_{11} + \theta_{12}) - l_b \quad (11)$$

$$\Gamma_{Y_3}(\theta) = 0 \quad (12)$$

For the case of a force acting in the same direction as the velocity of the body, the average power may be defined as the product of the force and the average velocity of the body, that is,

$$P_x = \frac{1}{T} \int_0^T \dot{x}_b^2(t) dt \quad (13)$$

$$P_y = \frac{1}{T} \int_0^T \dot{y}_b^2(t) dt \quad (14)$$

where \dot{x}_b^2 and \dot{y}_b^2 are related with the joint angular velocities by the following equations:

$$\dot{x}_b^2 = \sum_{i=1}^3 \Gamma_{X_i}^2(\theta) \cdot \dot{\theta}_{li}^2 + \sum_{j=1}^3 \sum_{k=1, k \neq j}^3 \Gamma_{X_j}(\theta) \cdot \Gamma_{X_k}(\theta) \cdot \dot{\theta}_{lj} \cdot \dot{\theta}_{lk} \quad (15)$$

$$\dot{y}_b^2 = \sum_{i=1}^3 \Gamma_{Y_i}^2(\theta) \cdot \dot{\theta}_{li}^2 + \sum_{j=1}^3 \sum_{k=1, k \neq j}^3 \Gamma_{Y_j}(\theta) \cdot \Gamma_{Y_k}(\theta) \cdot \dot{\theta}_{lj} \cdot \dot{\theta}_{lk} \quad (16)$$

The biped structure presents a nonlinear relationship between the input joint's rotation and the output's linear displacements. Consequently, some terms in the final Eqs. (13) and (14) are not of simple physical interpretation. The first term in summation represents the energy transferred by each individual joint (P_{x_i} and P_{y_i}). The second term corresponds to product velocity terms associated with energy coupling ($P_{x_{jk}}$ and $P_{y_{jk}}$). The purpose of the energy analysis method is to determine the input-output characteristics of the biped system and, if possible, to unmask its detailed structure. This involves finding the locomotion variables that move the locomotion system in a desired manner and to minimise the energy expended.

Perturbation analysis

In many practical cases the robotic system is noisy, that is, has internal random disturbing forces. As such, an approach called "perturbation analysis" was implemented to determine how the biped model has enough flexibility to match these distortions. First, the joint trajectories are computed by the inverse kinematics algorithm. Afterwards, the angular acceleration vectors are 'corrupted' by additive noise. For simplicity reasons, it is used a uniform distribution with zero mean added to the acceleration signal. As result, the trajectories of the body suffer some distortion and can only approximate the desired one. By regarding the forward kinematics of the mechanism, we can determine a two-dimensional index based on the statistical average of the well-known mean-square-error and defined by:

$$\xi_x = \frac{1}{N_S} \sum_{i=1}^{N_S} \sqrt{\frac{1}{T} \int_0^T [\dot{x}_b^i(t) - \dot{x}_b^d(t)]^2 dt} \quad (17)$$

$$\xi_y = \frac{1}{N_S} \sum_{i=1}^{N_S} \sqrt{\frac{1}{T} \int_0^T [\dot{y}_b^i(t) - \dot{y}_b^d(t)]^2 dt} \quad (18)$$

Here, N_S is the total number of steps for averaging purposes, \dot{x}_b^i and \dot{x}_b^d are the i th sample and the desired forward velocities at the body section, respectively, and y_b^i and y_b^d are the i th sample and the desired vertical oscillations at the body. In this method, the stochastic perturbation penalises the system's performance and we shall be concerned with minimising both indices ξ_x and ξ_y .

Lowpass frequency response

In robotics, the electro-mechanical system that regulates the movement (*e.g.*, actuators and drives) is constrained by its bandwidth. An important point to remember is that the bandwidth limits the maximum speed with which a system can respond to an input command. Hence, the practical conditions of motor control should also be considered when evaluating the system's performance.

In this perspective, the joint velocities are expanded in Fourier series and the time domain signals are described by the coefficients of their harmonic terms. Afterwards, these series are filtered through the same lowpass filter, which passes a certain portion of the frequency spectrum while neglecting all the higher harmonics. In doing so, the response of the system will differ from the input command, thereby producing a tracking error. Once again, we can compare the real and the desired trajectories produced at the body using, as our figure of merit, the mean-square-error as expressed in Eqs. (17) e (18) for $N_S = 1$. This method, based on frequency domain analysis, has also been successfully employed. However, it is a less general method than the perturbation analysis, because it requires the *a priori* knowledge about the nature of the robotic actuators.

Locomobility measure

The motivation that led to the development of the locomobility index was to apply the concepts of kinematic dexterity to biped walking. This performance measure can be expressed, in mathematical terms, using the geometrical representation of the kinematics through the Jacobian matrix (Park, 1994). In our case, we selected a two-dimensional index of the global directional uniformity of the motion based on the projections of the major axis of the ellipsoid.

The global index is obtained by averaging the longest projections of the major axis along the x -axis (E_{P_x}) and the y -axis (E_{P_y}), over a complete cycle:

$$L_x = \frac{1}{T} \int_0^T E_{P_x}(t) dt \quad (19)$$

$$L_y = \frac{1}{T} \int_0^T E_{P_y}(t) dt \quad (20)$$

The more suitable trajectory is that which maximises L_x and minimises L_y .

Simulation Results

In this section, we perform a set of experiments to estimate how the biped robot adapts to variations in the locomotion variables. The aim is to determine the main factors that optimise the leg motion and to compare the formulated dexterity measures. Each places an emphasis on distinct objectives and each introduces different mathematical formulations. Yet, as this section is intended to suggest, there is a correlation among such apparently different approaches. To evaluate the system's operating features, the simulations are carried out considering a forward velocity $V_F = 1$ m/sec.

Step Length and Hip Height

To illustrate the use of the preceding results, the biped locomotion was simulated. At this phase, a major simplification is introduced by allowing the swing foot stays on the ground for all time without violating any constraint. The individual indices are computed while maintaining a constant hip height during the stride. As shown in Fig. 4, the performance function is evaluated with respect to the step length and the hip height (for equal link lengths). In this case, we are plotting the average power in

terms of x and y coordinates. For convenience, the surface portion corresponding to hip height values less than 0.5 m is not represented.

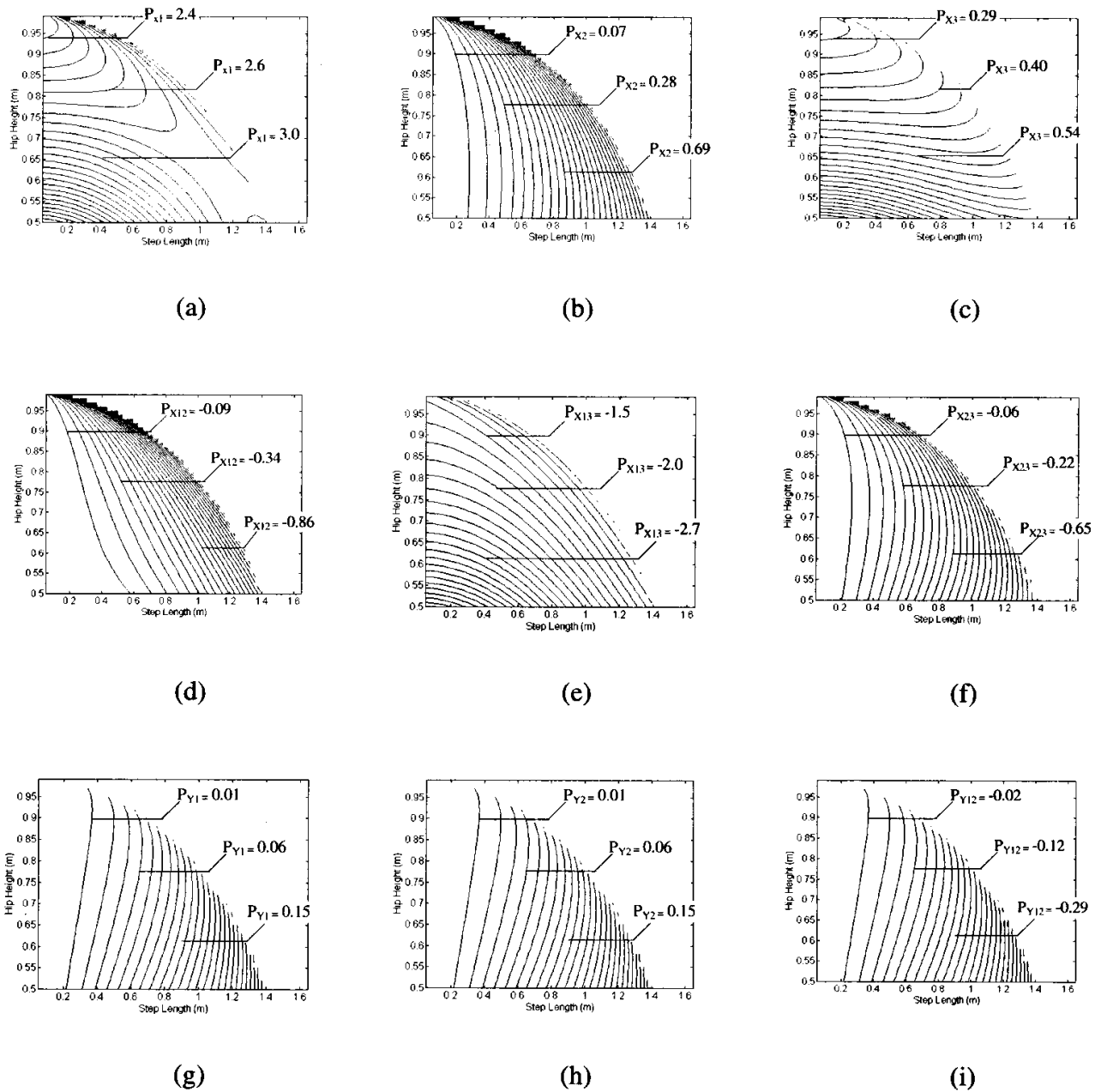


Fig. 4 Contour plot of the average power along a complete cycle of walking

The ankle joint is believed to dominate the process of walking, as seen from the energy values involved in every joint (Figure 4(a-c)). The simulation, also, indicates that the knee joint may be a major factor due a significant range of performance variation. As shown in Figs. 4(b) and 4(h), an important degradation occurs as the step length value increases. On the other side, the hip seems to be particularly sensitive to the conditions of hip height (Figure 4(c)). In what concerns the cross coupling terms (Figures 4(d-f) and 4(i)), the average power along a complete cycle resulted in negative values.

Figure 5(a) corresponds to the perturbation analysis method. Considering these plots, we conclude that an important degradation occurs as the step length increases. The hip height variable also affects the measure, however, has a smaller influence. In Figure 5(b), we depict the results of applying the locomobility index. The basic idea to retain is that to maximise L_x and, additionally, to minimise L_v , the hip height is expected to round about 95% of the maximum height.

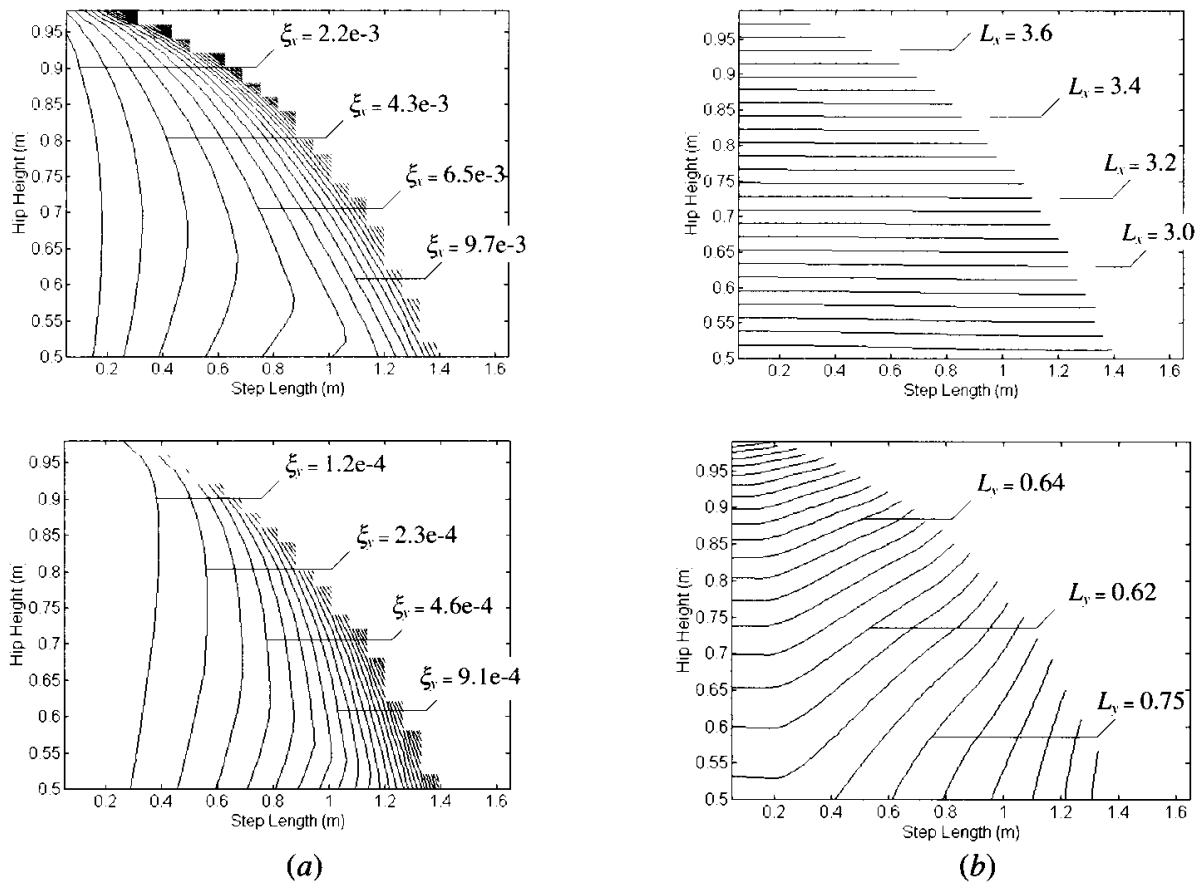


Fig. 5 Contour plot of the performance surface along the x and y-axis: (a) perturbation analysis; (b) locomobility index

Finally, let us examine the “lowpass frequency response” method. The fundamental harmonic f_0 of the waveforms at the joints is given by,

$$f_0 = \frac{V_F}{2L_S} \quad (21)$$

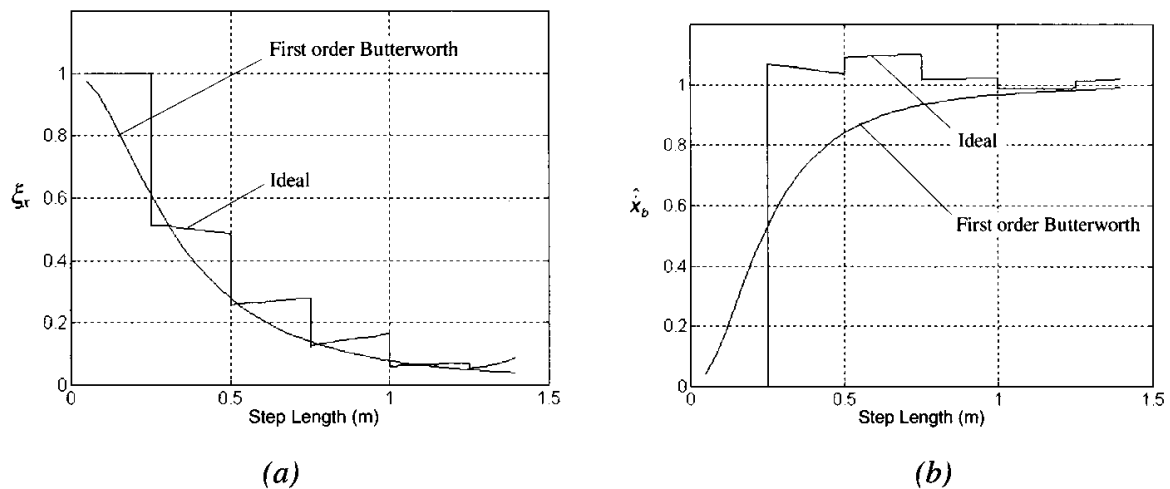


Fig. 6 Lowpass frequency response at the body: (a) mean-square error (ξ_x); (b) mean forward velocity (\hat{x}_b).

In Figure 6(a) we have used both an ideal and a Butterworth lowpass filter with bandwidth $f_c = 2$ Hz. With an ideal lowpass filter, the performance surface presents a set of discontinuity points

that solely depend on the number of harmonic terms up to f_c . Beyond the mean square error along the x -direction ξ_x , we have also analysed the influence of the bandwidth on the mean forward velocity \dot{x}_b (see Fig. 6(b)). In contrast to the other methods, the results suggest that the performance improves for higher step lengths. Both indices (ξ_x and \dot{x}_b) remain almost constant for different hip heights. In order to achieve broader applicability, we must provide information about the robotic actuators by means of a mathematical model (e.g., frequency response).

Hip Ripple

Upon this point, we consider a hip trajectory with a sinusoidal oscillation. In this case, the foot of the swing leg remains at the ground for all cycle. The contour plot in Fig. 7 displays the performance values using the perturbation analysis method. From this simulation, we can observe that the optimum hip ripple tends to a zero value and, furthermore, it remains almost unchanged to hip height variations.

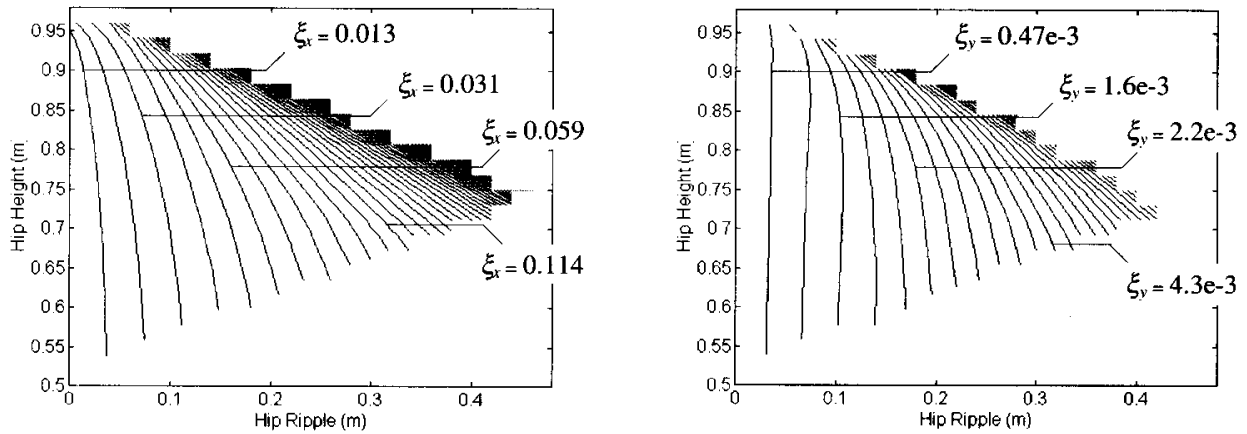


Fig. 7 Contour plot: H_H versus H_R ($L_S = 40$ cm)

Foot Clearance

Another variable that should be tested is the effect of foot clearance. Until now, all the experiences considered that the foot stays on the ground without any restriction. Next, the goal is to analyse the situation in which the foot can be lifted off the ground. In Fig. 8 we study the influence of F_c for different hip height conditions. In kinematic terms, the minimum foot clearance, when avoiding an accidental contact, is the optimal one. We confirmed this property of the foot clearance using the locomobility analysis.

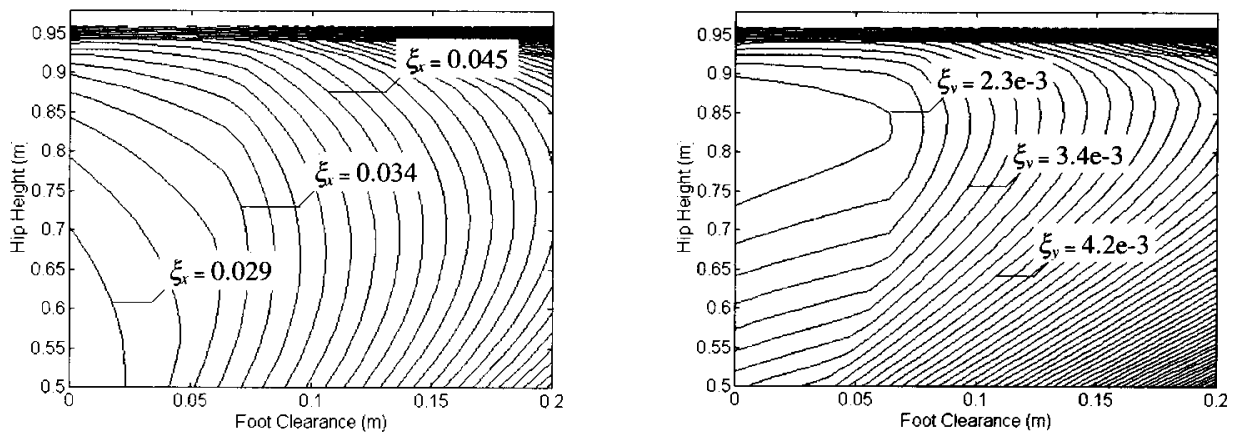


Fig. 8 Contour plot: H_H versus F_C ($L_S = 40$ cm).

Relative Link Lengths

We now investigate the role of the link lengths, l_1 and l_2 , in the system's performance. The choice of the leg lengths is a relevant aspect at the design process and affects the robot's mobility, as well. Figure 9 depicts the curve of ξ_x given the link length l_1 , when employing the perturbation method. As can be observed, the optimum solution occurs when the knee-ankle length is higher ($l_1 = 0.74$ cm) than the hip-knee length ($l_2 = 0.26$ cm). The simulation also indicates that for l_1 in the range from 40 to 90 cm the performance index remains almost constant. The variation found along the y-axis is insignificant and is not represented.

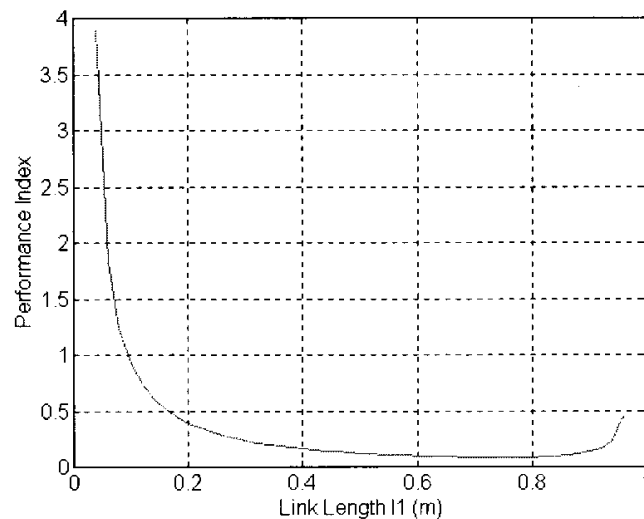


Fig. 9 Performance curve versus link length l_1

Conclusions

Nowadays, robots are widely used in the manufacture industry. However, the applications of these industrial robots are relatively simple and, for the most part, involving repetitive tasks previously performed by human beings. Bringing robots out of engineered environments and striving for increased autonomy, reliability and dexterity, is both an enormous challenge and a socio-economic necessity in the near future. One key component required to build such intelligent systems is the locomotor system. Accordingly, the biped locomotion aspects of robotics are gaining more and more attention in both research and applications.

In this paper, we have compared various kinematic aspects of biped locomotion. By implementing different motion patterns, we estimated how the robot responds to a variety of locomotion variables such as hip ripple, hip offset, foot clearance and relative link lengths. Various quantitative measures were formulated for analysing the kinematic performance of such systems. Given these goals, the graphical results provide a more concrete illustration of the system's properties. The energy analysis method has the advantage that is very amenable to computer implementation and, furthermore, it is computationally more efficient. However, this approach has the drawback that the resulting equations are not in a completely intuitive form. In general, the perturbation analysis method tends to be more elegant although computationally more exigent. Its random characteristic seems to be particularly tailored for examining the role of the different variables on the locomotion process. On the other hand, the results obtained using the locomobility measure present a correlation with that of the perturbation analysis. The lowpass frequency response produces basically different effects allowing a different perspective of the problem. Moreover, some knowledge of the actuator's characteristics will always be helpful when evaluating the system's performance.

While our focus has been on kinematic dexterity, certain aspects of locomotion are not necessarily captured by the measures proposed. Thus, future work in this area will address the refinement of our models to incorporate dynamics, as well as exploring human-like walking principles. In contrast to

rotational actuators used in robotics, the skeletal is completely controlled by linear actuators organised in agonist-antagonist pairs. A complementary analysis could reveal some important properties of both approaches giving some hints to understand the performance differences between robotic and biological motion.

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