

Comparative study about heating systems for pultrusion process

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ABSTRACT

Different heating systems have been used in pultrusion, where the most widely used heaters are planar resistances. The primary objective of this study was to develop an improved heating system and compare its performance with that of a system with planar resistances. In this study, thermography was used to better understand the temperature profile along the die. Finite element analysis was performed to determine the amount of energy consumed by the heating systems. Improvements were made to the die to test the new heating system, and it was found that the new system reduced the setup time and energy consumption by approximately 57%.

Keywords: Glass fibres, Finite element analysis, Pultrusion Conserving energy

1. Introduction

Pultrusion is a continuous process employed worldwide to manufacture polymer composites into shapes with constant cross sections, such as tubes, bars and others profiles [1–4]. Polymer composites, which are made from a thermoplastic or thermosetting resin matrix and glass fibre reinforcements, are generally referred to as glass fibre-reinforced plastic (GFRP) composites [5] and are used in structural applications, where a high specific mechanical strength and good chemical resistance are required [6]. Because GFRP composites are light-weight, they have a strong competitive advantage: their structures are easy to transport, which is extremely important in specific situations. Their assembly requires rivets, adhesives, and screws, among other common parts. Being light-weight and requiring simple mechanical fasteners is advantageous in structures, such as wastewater treatment plants, structures near salt water environments, and chemical storage tanks. The tool die, which is used to create the final product shape, is usually divided in two parts, the upper and lower part. When the cross sections of the pultruded shapes have openings, such as tubes, mandrels are required [5]. The main advantages of this process are related to its low labour requirements and tooling simplicity.

To better conform the product to its final application, appropriate materials and parameters must be selected. Composites can be reinforced with fibres that are unidirectional, matted or woven.

The usage of surfacing veils must be considered if good surface quality is required [7].

Epoxy, polyester and vinyl esters are the most commonly used resins. Epoxies are better adhesives, generally adhering to the die better than polyester and vinyl esters; however, this leads to more manufacturing problems due to the composite breaking while in the die. Moreover, this type of resin shrinks less, which worsens the problem [8]. However, composites made from epoxies are extremely advantageous when special mechanical properties, elevated thermal resistance and high corrosion are required.

After the fibres are impregnated into the thermosetting resin and while still in the uncured state, they are guided through tools that organise and pre-form the fibres into the right position; also, any excess resin is squeezed out, which is known as “debulking”. To enhance the structural properties and surface finish, a strand mat and surface veils are often added at this stage. The curing process begins when the composite passes through a heated die.

The die length is one of the most important aspects in pultrusion because it determines how long the composite will be heated. The die length will also affect the pulling force and production line speed due to the variation of the inside surface area. A programmed power controller controls the temperature profile along the die, recording real-time measurements using temperature probes and providing the appropriate temperature profile for the thermosetting resin cure process. The temperature profile along the die is important because if the temperature is too low, the resin will not completely cure, and if the temperature is too high, the resin degrades, which creates manufacturing problems; the most critical problem is when the moulded profile breaks while inside

the die [7,9]. Depending on the product being manufactured, variables such as the type of matrix resin and its exothermic reaction, line speed, temperature along the die, ply schedule and die length must be appropriately adjusted to optimise production [7]. The pultrusion process is often decided by tests during manufacturing, which allow producers to observe the relations between the variables and therefore increase the parameter accuracy related with each product. When the product exits the die, its profile is cured and only needs to be cut to the desired length.

In this process, a dual traction system is used, which continuously pulls the GFRP composite through the die [1,5]. The pulling speed depends on several conditions, such as the bar thickness, die length, die temperature and resin formulation. To incorporate all these conditions, experience is fundamental to achieve the optimal speed with high quality standards. At the end of the process, the pultruded composite is cut to the required length dimensions [1–3].

Generally, the heating system efficiency is not an important issue in the pultrusion process. This fact is supported by the widespread use of external heating systems [10,11] that are associated with significant heat losses. However, due to global warming, it is imperative to develop a more efficient heating system, which would lower the product cost and reduce environmental impacts. The typical die temperature profile is shown in Fig. 1. The exothermic reaction causes the maximum temperature to be reached, approximately midway through the length of the die; the temperature then drops, reaching an exit value at the end of the die, in this case [1].

As mentioned before, the most common heating systems used in the pultrusion process are the large universal heating platens and the multi-planar heaters. These systems have low setup times but have large heating losses. Thus, other systems have already been suggested to reduce the heating losses, such as fluid circulation and electrical cartridges heaters. The fluid circulation solution will increase the setup time and requires a pump and will consequently consume electricity, which will result in more maintenance compared with other solutions [11]. Therefore, the electrical cartridge heating system was chosen for this study due to its higher energy savings and lower setup time and because it allows temperature refinement in different zones along the die.

This work was greatly supported by finite element analysis, which has also been used by other researchers [10–14] due to the expensive costs associated with experimental tests. In this study, thermographic images of the die with planar heaters were obtained and analysed. Then, the heating system was simulated, and the results were matched with the thermographic images. Afterwards, it was possible to verify that the simulations, along with several assumptions, can provide results that are acceptable.

2. Experimental

Fig. 2 shows the pultrusion system layout, and Fig. 3 shows the heating system used for the die at the beginning of the study.

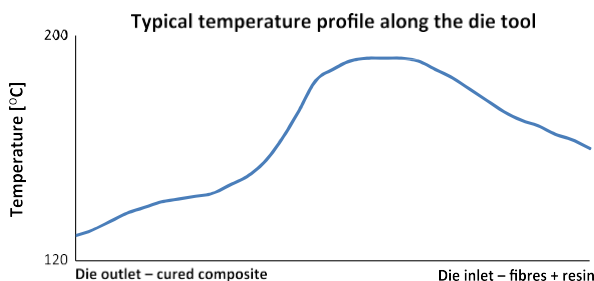


Fig. 1. Typical temperature profile along the die tool for the resin formulation used.

The heating system consists of four aluminium heaters, which were formed by Cr–Ni spiralled wire embedded in a cast aluminium block, and divided in two groups, “a” and “b”, where each group consisted of two heaters. In each group, one heater was placed on the die and the other heater was placed under the die. The length, width and height of the group “a” heaters were 350 mm, 70 mm and 30 mm, respectively, and the length, width and height of the group “b” heaters were 380 mm, 140 mm and 30 mm, respectively. Two temperature sensors, one for each group of heaters (group “a” and group “b”), sent real-time information to a PLC (programmable logic controller), which monitored whether the temperature stayed within the limits. If the value was lower than a minimum value, then the heaters turned on, and if the value was greater than a maximum value, a turn-off command was sent. The heating system had four aluminium rectangular resistances, each 800 W. Because all the heaters had the same power, the size difference and the consequent variation in volume only influenced the ratio, W/m³. Clamps were used to connect the heaters to the die.

The pultrusion process consists of many parameters that must be controlled. The following are the most common parameters used by the manufacturer for the “U” profile:

- Line speed: 50 cm/min
- Fibre type: glass fibre type E
- Number of fibre wires: 34 with 4800 TEX
- Resin/fibre ratio: 30/70
- Resin type: unsaturated polyester resin with low viscosity
- Resin gel time to exothermic peak: 2 min
- Exothermic peak: 230 °C
- Composite tensile strength (mean): 370 MPa
- Barcol hardness: 45
- Normal production per hour: 30 m

The length, width and height of the die tool used were 900 mm, 103 mm and 56 mm, respectively. The width, height and thickness of the internal “U” shape were 51 mm, 12 mm and 4 mm, respectively. The fillet had a radius of 1 mm. All the internal surfaces had an electroplated hard chromium surface treatment that were polished until a roughness of 0.08 μ m was achieved.

Thermographic images were captured to compare the experimental results with the simulation results. These images were taken during a stable manufacturing state, which allowed the temperature profile along the die to be obtained.

The thermographic images were taken using Flir®i40 imaging camera that had the following characteristics:

- Spectral band: large wave (7.5–13 μ m);
- Detector type: focal plane array (FPA) microbolometer 120x 120 pixels;
- Frame rate: 9 Hz;
- Accuracy: ± 2 °C or $\pm 2\%$ of the reading;
- Thermal sensitivity: < 0.20 °C to $+25$ °C;
- Range of object temperatures: -10 to $+350$ °C;
- Display: 89 mm colour LCD, 18 bit colour;
- Interpolation: imaging detector interpolated to 240 x 240 pixels.

The measurement procedures followed the equipment supplier specifications: a 1 m distance was maintained between the camera and the die and their alignment was kept. The emissivity of the die material was inputted into the thermographic equipment, which resulted in more accurate results. The thermographic images were analysed using the program provided by the equipment supplier (Flir® Quick Report). Each image was divided into 100 different areas corresponding to a matrix with four rows and 25 columns, which represented the lateral surface of the die, as shown in Fig. 4.

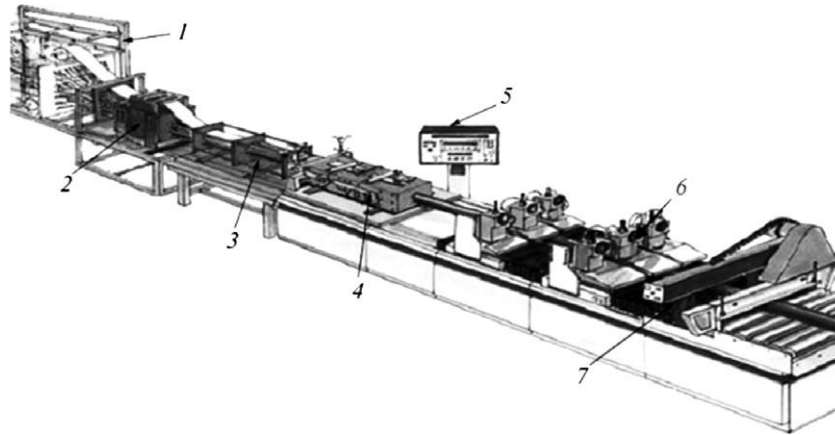


Fig. 2. Pultrusion layout (adapted from [7]), where the following can be observed: (1) fibres and roving rack, (2) impregnator, (3) folder, (4) die, (5) control board, (6) withdrawal roll facility and (7) cutting device.

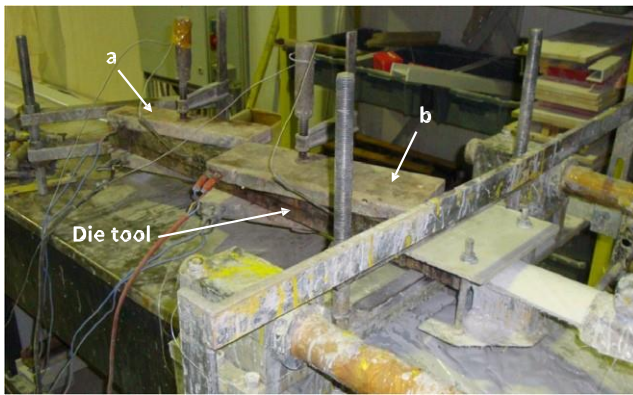


Fig. 3. Initial set-up die with 800-W plane heaters: (a) small and (b) large electrical resistance.

Following each session, the images were analysed using the aforementioned software, which determined the temperature at each point corresponding to the intersection of the horizontal dashed lines with the vertical lines in Fig. 4. To increase the image definition, measurements were taken from two die sections, referred to as a single session. Three sessions were performed over the die lateral wall to increase the accuracy of the results. After all the images were analysed, statistical tools were applied to the images, which allowed the measured temperature profile along the die to be used as a reference for further developments.

To perform the Finite Element Analyses (FEA), the SolidWorks® software was used to model the die tool/heaters system. The die tool was modelled as upper and lower parts, and the heaters were mod-

elled as independent bodies. The thermal state was assumed to be transient, and the following were assumed: all contacts were considered perfect; all bodies were initially at room temperature (22 °C) and remained constant; the convection coefficient was 2 W/m² °C; the emissivity was 0.8. To increase the software response, a tetrahedral (four nodes) mesh was used, which, after convergence, resulted in 256,975 elements. Specialised support literature was used as a reference to gain a deeper understanding regarding the procedures and methods used for this type of analysis [15].

This method was used to evaluate the temperature fluctuation in the die, which allowed the determination of the time the resistances needed to be connected to the power supply. System “a” and “b” (see Fig. 3) were independently controlled. The points chosen for the temperature control in the simulation corresponded to the locations where the temperature probes were located on the real die. These control points were located 180 mm and 684 mm from the exit side of the die. Then, the temperatures at these points were checked with the drawn temperature profile and used as a reference to perform the FEA. A temperature difference of $\Delta T = \pm 5$ °C is typically used by the manufacturer for the “U” profile, which means that the power supply switches on or off when the temperature is 5 °C below or above the reference temperature, respectively. The simulation ran for 4500 seconds. While the simulation was developing and the results were being analysed, it was observed that the die required approximately 3600 seconds to warm up (the goal was a stable temperature behaviour). However, beyond this warm-up period, a period of 900 seconds was simulated, a time that was considered as a representative period that would be repeated in a typical work day for a die.

At the end of the simulation, a temperature profile was obtained and compared with the profile obtained by thermography. More-

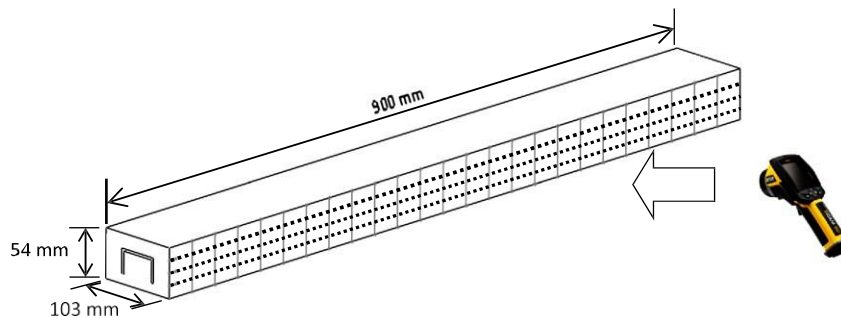


Fig. 4. Schematic representation of all the points considered on the lateral wall of the die that were analysed by thermography.

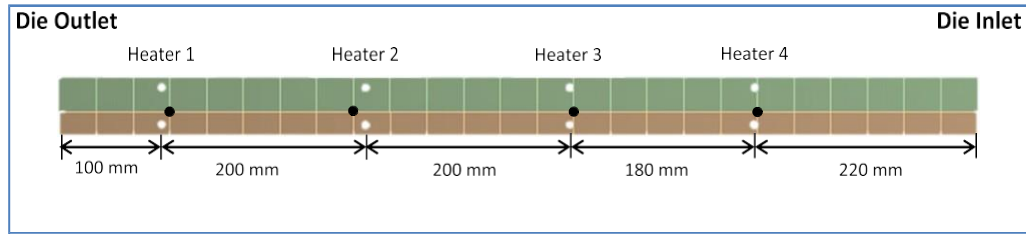


Fig. 5. Location scheme of the resistance pairs (white spots) and temperature probes (black spots) along the die tool (lateral view).

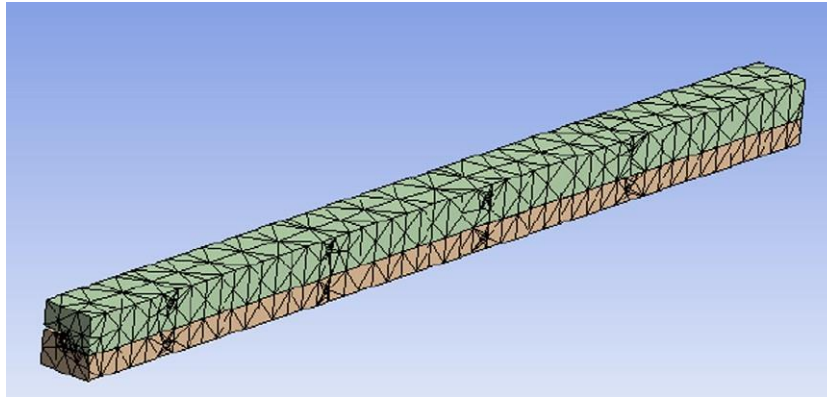


Fig. 6. Half of the die tool with its mesh, which was done to reduce the computational time.

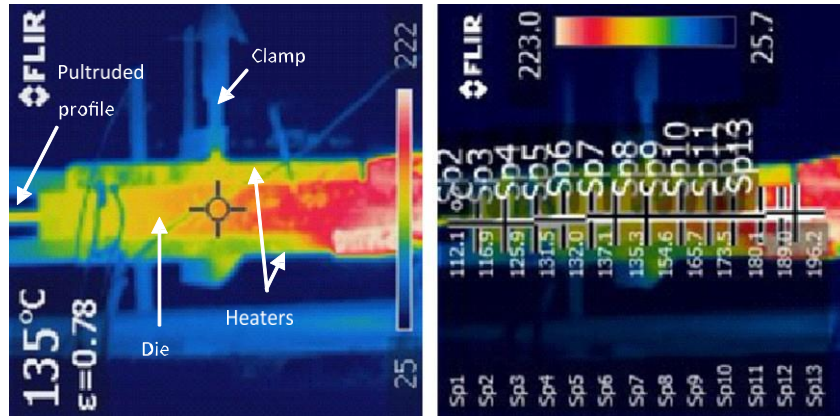


Fig. 7. Thermographic images with and without the analysis of the previous heating system.

over, this simulation allowed the heaters' working time to be studied.

Several weaknesses were observed in the heating system, which can be easily solved by inexpensive solutions. The set-up and the assembly method can be improved. It is difficult for an employee to single-handedly attach each heater group to the die using a clamp. Moreover, using only one clamp at the heater centre, as shown in Fig. 3, causes the heaters to bend, which decreases the conduction heat transfer due to the lack of contact. Moreover, only one of the six surfaces of the heater is in contact with the die, and thus, much of the heat does not go to the die. To overcome this problem, the existing holes in the die were used to modify the system, where cylindrical heaters were embedded into the die, which has been performed by Sumerak [11]. The PLC of the pultrusion machine is capable of controlling up to four channels and thus, the previous heating systems ("a" and "b") were replaced by

four independent heating systems, which included a temperature probe, allowing a more refined control of the temperature. The power used to heat the die tool is the same as before; each cylindrical heater had a rated power of 400 W. The objective of studying this system was to reduce the set-up time and improve the thermal efficiency. Fig. 5 shows the layout of the resistances along the die that were inserted into the existing holes, which were where the previously used temperature probes were inserted when the planar heaters were used. These holes were enlarged by drilling so the cylindrical resistances could be inserted. The new system was simulated with the same assumptions used previously. The temperature was controlled at the black spots near each pair of heaters (see Fig. 5), using the limits, $DT = \pm 5 \text{ }^{\circ}\text{C}$, which were also used in the previous system. The reference temperature for each one of the four points was obtained by the thermographic temperature profile, which was also used for the first system.

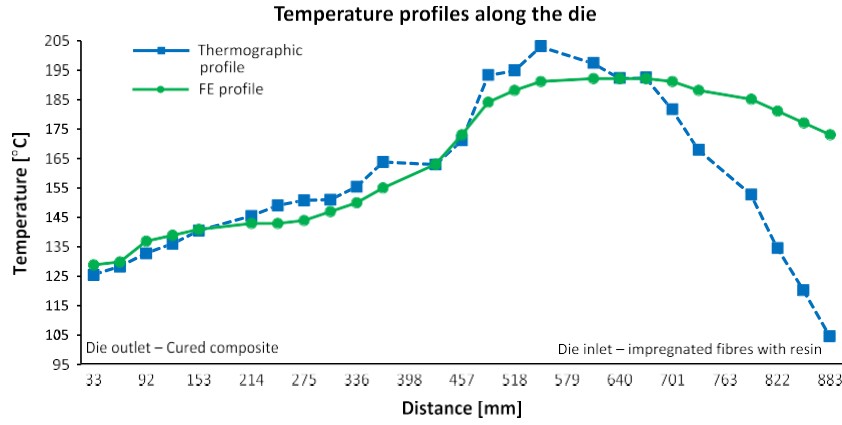


Fig. 8. Thermographic and FEA temperature profiles.

The tetrahedral (four nodes) mesh was used, which after convergence, resulted in 11,089 elements. Only half of the die tool was simulated due to symmetry along the die length, as shown in Fig. 6, which reduced the computation time.

3. Results and discussion

The model was based on the thermographic analysis, as shown in Fig. 7, which depicts the obtained temperature profile study along the die. As previously mentioned, the thermographic measurements on the die were performed in two parts: one image for each half of the die. Each session consisted of these two images. This method resulted in more accurate results during the image analysis. Three sessions were performed on three different days under similar environmental conditions; all sessions were conducted during manufacturing time to ensure the warm-up periods were always avoided. The temperature could be seen for each selected point. These points were obtained by intersecting the horizontal dashed lines with the vertical dashed lines, as shown in Fig. 4. Statistical tools were applied to all the obtained images. For each image, an average temperature was collected at the points on the same vertical line. Then, the results from all the images were averaged to obtain the temperature profile along the die, as shown in Fig. 8.

The FEA was performed using the following method: first, all the heaters were turned on for a period of time and then, the temperature at each point was checked. It was observed that the points located 180 mm and 684 mm from the exit side of the die reached the desired temperature more rapidly. Controlling these two points is required to avoid overheating, which can degrade the composite. Thus, these points were selected to be temperature controllers for each heating system, “a” and “b”, respectively. According to the temperature profile obtained by thermography (Fig. 8), the reference temperatures of groups “a” and “b” were $141 \text{ }^{\circ}\text{C} \pm 5 \text{ }^{\circ}\text{C}$ and

$196 \text{ }^{\circ}\text{C} \pm 5 \text{ }^{\circ}\text{C}$, respectively. A temperature range of $\pm 5 \text{ }^{\circ}\text{C}$ was selected based on knowledge from the manufacturer.

At the end of the simulation, two different approaches were then used. First, the agreement between the temperature profile drawn by thermography and the one obtained by the FEA was verified, as shown in Fig. 8. The maximum deviation obtained was 7%, which was considered acceptable for this analysis. On the right side of Fig. 8, a large disagreement between the curves can be observed; however, this was due to operational reasons. Following the correct procedures for thermographic measurements, this part of the die tool is located behind the fastening elements that are used to fix the die to the pultrusion equipment, which blocks the camera’s infra-red beam. The right side of the curve (Fig. 8) corresponds to the die entrance (resin plus the fibres), whereas the left side corresponds to the exit side (cured pultruded product). The current temperature profile could be drawn because 3600 s had passed; at this time, the temperature is stabilised along the die, subject only to small variations. Then, a study on the power used by the heaters during an entire working day was performed, as shown in Table 1. This table was developed with information on the heaters’ “turn on” time. With the total working hours for each group of heaters and their power known, the amount of kWh used could be determined. Fig. 9 shows graphically the information used. The continuous line denotes the small heaters, and the dashed line denotes the large heaters. It is important to note that the simulation of the new heating system, based on eight cylindrical heaters embedded in the die tool, will follow the same temperature profile obtained by the thermography. The study will focus on the working time of the heaters and compare the time with that of the previous heating system.

After the FEA was performed, Table 2 was created to show detailed information regarding each group of heaters. To better compare the results between the old and new system, Table 3 was created. The subtotal working time (SWT) was calculated using the following equation: $\text{SWT} = \text{Warm-up time} \times \text{kWh} + \text{Manufacturing time} \times \text{kWh}$. In this table, the working times of heating systems 1 and 2 from Fig. 5 are presented together to compare them with that of heating system “a” from Fig. 3, and the results

of heating systems 3 and 4 from Fig. 5 are shown together to compare with that of heating system “b” from Fig. 3. In Figs. 10–13, the temperature performances for each heater are shown. Comparing the data from Table 1 and Table 3, the difference between both heating systems can be analysed, as shown in Table 4.

The reduction in the energy consumed with the new system was approximately 57%, which is significant. As more thermography sessions were performed and while testing the cylindrical heating system during manufacturing, it is possible to compare

Table 1
Detailed power consumption of the planar heaters.

Planar heaters			
	Warm-up (1 h) (kW h)	Manufacturing (7 h) (kWh)	Subtotal working time (8 h) (kW h)
Small heaters	6.6E^{-1}	19.2E^{-2}	2.0
Large heaters	141.2E^{-2}	94.8E^{-2}	8.0
Total power consumption per manufacturing day (kW h)			10.0
Total power consumption per manufacturing month (22 days) (kW h)			221.1

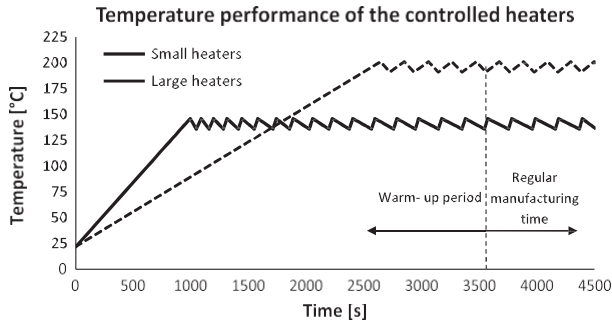


Fig. 9. Temperature performance over time of the controlled heaters.

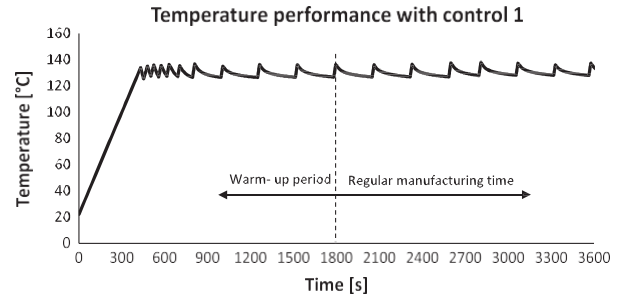


Fig. 10. Temperature performance over time with control 1.

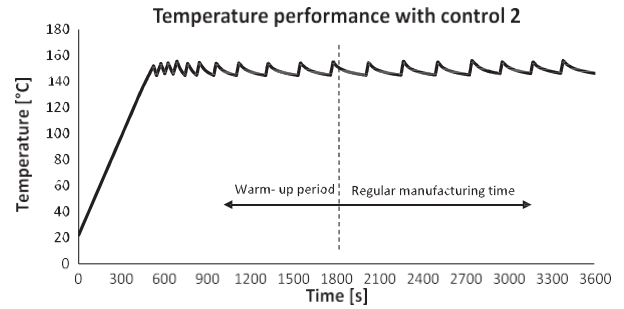


Fig. 11. Temperature performance over time with control 2.

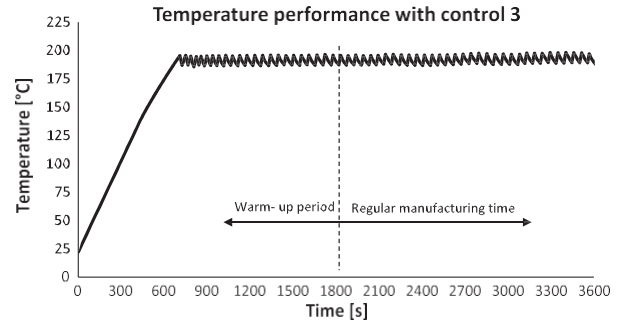


Fig. 12. Temperature performance over time with control 3.

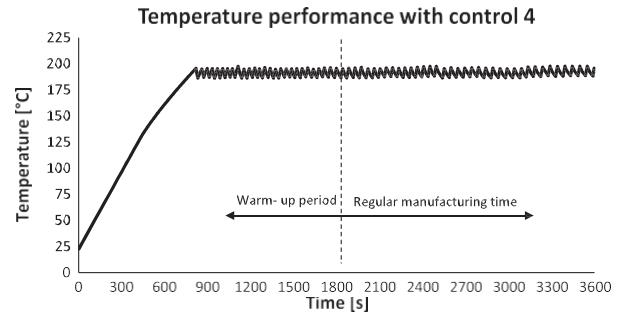


Fig. 13. Temperature performance over time with control 4.

Table 2
Detailed results of the power consumption for the cylindrical heating system.

Heater 1		Heater 2	
Warm-up (h)	0.5	Warm-up (h)	0.5
Total time of work in 30 min. (min)	9.650	Total time of work in 30 min. (min)	11.700
Total time of work during 1 h (h)	0.322	Total time of work during 1 h (h)	0.390
Heat power (W)	800	Heat power (W)	800
Power usage (kW h)	0.258	Power usage (kW h)	0.312
Manufacturing time (h)	7.5	Manufacturing time (h)	7.5
Total time of work in 30 min. (min)	1.400	Total time of work in 30 min. (min)	2.050
Total time of work during 1 h (h)	0.047	Total time of work during 1 h (h)	0.068
Heat power (W)	800	Heat power (W)	800
Power usage (kW h)	0.038	Power usage (kW h)	0.054
Heater 3		Heater 4	
Warm-up (h)	0.5	Warm-up (h)	0.5
Total time of work in 30 min. (min)	16.620	Total time of work in 30 min. (min)	20.180
Total time of work during 1 h (h)	0.554	Total time of work during 1 h (h)	0.673
Heat power (W)	800	Heat power (W)	800
Power usage (kW h)	0.443	Power usage (kW h)	0.538
Manufacturing time (h)	7.5	Manufacturing time (h)	7.5
Total time of work in 30 min. (min)	6.720	Total time of work in 30 min. (min)	7.720
Total time of work during 1 h (h)	0.224	Total time of work during 1 h (h)	0.257
Heat power (W)	800	Heat power (W)	800
Power usage (kW h)	0.179	Power usage (kW h)	0.206

Table 3
Results of power consumption for the cylindrical heating system.

Cylindrical heaters			
	Warm-up(0,5 h) (kW h)	Manufacturing (7,5 h) (kW h)	Subtotal working time (8 h) (kW h)
Heating systems 1 and 2	$56.9E^{-2}$	$9.2E^{-2}$	$97.5E^{-2}$
Heating systems 3 and 4	$98.1E^{-2}$	$38.5E^{-2}$	$337.7E^{-2}$
Total power consumption per manufacturing day (kW h)	$435.2E^{-2}$		
Total power consumption per manufacturing month (22 days) (kW h)	$9574.4E^{-2}$		

the thermographic images with the thermographic images from the previous system. Fig. 14 shows the same zone of the die tool heated by the two different systems, where “A” corresponds to the planar heater and “B” corresponds to the cylindrical heaters.

Table 4
Comparison between the planar heaters and cylindrical heaters.

Planar heaters – power consumption per manufacturing day (kW h)	10.0
Cylindrical heaters – power consumption per manufacturing day (kW h)	4.4
Reduction (%)	–56.7

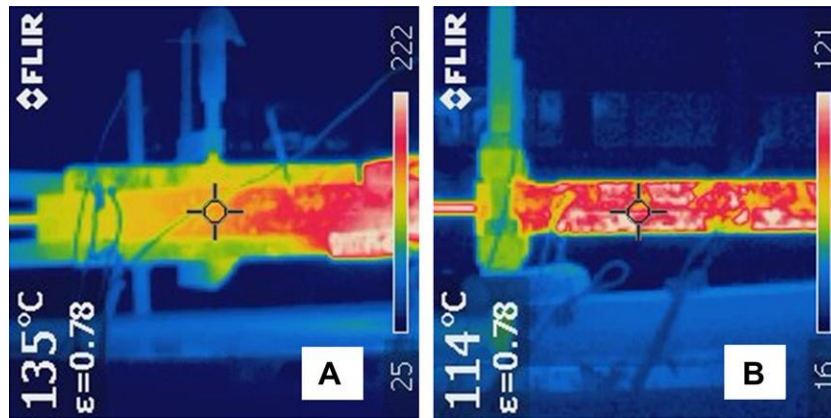


Fig. 14. Thermographic images showing the differences in the homogeneity of the temperature. A – planar heaters. B – cylindrical heaters.

Besides being more efficient, the system with the cylindrical heaters resulted in a more homogeneous heat flow and better control of the temperature for each die zone compared with that with the planar heaters.

4. Conclusions

The primary objective of this work was to reduce the energy consumption of the pultrusion process by creating a more efficient heating system. Using technologies such as thermography in the temperature analysis and the finite element method to determine how the heat flows in each heating system, experimental work was performed, which verified the following:

- Thermographic images and use of the corresponding software analysis allowed the temperature profile along the die to be determined;
- The temperature along the initial die followed the usual profile exhibited by other similar dies;
- The finite element method was used to compute how much time the resistances were connected to the power supply, which allowed the energy consumption of the system to be determined.

Thus, the following conclusions can be drawn:

- The temperature profile with the new heating system remained identical to that of the older heating system;
- The power consumption decreased 57% using the embedded cylindrical heaters compared with that of the initial system, where the resistances were located in existing holes in the die;
- The warm-up time was reduced up to 50%, improving the set-up time and increasing the production time;
- Temperature control was more refined due to the increased number of temperature probes, which allowed better refinements if necessary.

Moreover, this new heating system was tested during manufacturing and maintained high-quality standards without manufacturing problems. Thus, this system is highly suggested to increase productivity, maintain a high quality of the pultruded products and to reduce the set-up and warm-up times as well as reduce the energy consumption. However, further work must be conducted to optimise the resistance position.

Acknowledgements

The authors would like to thank QREN – Quadro de Referência Estratégico Nacional for their financial support and ISEP – Instituto Superior de Engenharia do Porto for allowing us to use their facilities and equipment. Last, but not least, the authors would like to thank ALTO–Perfis Pultrudidos, Lda. for all the experimental work performed on their pultrusion machine. Without their help, this research could not have been finished.

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