

Fractional model for malaria transmission under control strategies

Carla M.A. Pinto, J.A. Tenreiro Machado

A B S T R A C T

We study a fractional model for malaria transmission under control strategies. We consider the integer order model proposed by Chiyaka et al. (2008) in [15] and modify it to become a fractional order model. We study numerically the model for variation of the values of the fractional derivative and of the parameter that models personal protection, b . From observation of the figures we conclude that as b is increased from 0 to 1 there is a corresponding decrease in the number of infectious humans and infectious mosquitoes, for all values of α . This means that this result is invariant for variation of fractional derivative, in the values tested. These results are in agreement with those obtained in Chiyaka et al. (2008) [15] for $\alpha = 1.0$ and suggest that our fractional model is epidemiologically well-posed.

Keywords:

Malaria transmission
Fractional mathematical model
Delay differential equations

1. Introduction

Malaria is a major problem in tropical and subtropical regions around the planet. Malaria increases morbidity and mortality in these regions, causing an enormous impact in their health systems and economies. The World Health Organization (WHO) reported that malaria alone, or in combination with other diseases, kills, every year, approximately 1.1 to 2.7 million people all over the world; 3.3 billion being at high risk of malaria. Sub-Saharan Africa is the most affected region with one million deaths estimated annually [1].

Reducing the spread of malaria, with the implementation of selective and sustainable preventive measures, is extremely important. These measures should halt the deterioration of the malaria situation. Insecticide-treated materials, such as mosquito nets, have been successfully and safely used to control the malaria epidemic in Africa and Western Pacific regions. Nevertheless, records show that the anopheline mosquitoes has started to resist the insecticide (pyrethroids), used to treat the nets. Other measures are the indoor residual spraying with (DDT) insecticides, the insect repellent creams, that are also very effective. Nevertheless, the creams are more expensive than the nets. In 2012, Beier et al. [2], proposed attractive toxic sugar bait methods (ATSB) to control *Anopheles gambiae* mosquitoes in the arid malaria-free oasis environment of Israel. Authors found that ATSB methods, for malaria vector control, are highly effective in arid environments, despite the presence of competitive, highly attractive natural sugar sources. The draining of wetlands and other standing waters is also an effective public health measure [3]. Chemoprophylaxis is used as a preventive measure in malaria-endemic regions, specially in pregnant women. It helps to decrease the incidence of low birth weight and severe maternal anemia. Nevertheless, chemoprophylaxis is weakly effective due to lack of patient compliance and drug resistance of *P. falciparum*. The role of gametocytocidal drugs, in cases of low transmission, may be significant in preventing malaria. It reduces gametocyte counts in infected patients, decreasing the rate of transmission. Transgenic mosquitoes, when available, may offer new advantages in preventing malaria.

Another extremely significant preventive measure is malaria vaccination. An enormous effort has been made in the context of developing an effective vaccine, over the last few decades. In 1988, Patarroyo et al. [4] developed a synthetic vaccine to protect humans against *Plasmodium falciparum*. The SPf66 vaccine is a synthetic hybrid peptide polymer containing amino acid sequences derived from three *P. falciparum* asexual blood stage proteins, linked by repeat sequences from a protein found on the *P. falciparum* sporozoite surface [5]. SPf66 has undergone extensive field tests in endemic malaria areas. More recently, new candidates to vaccines, such as RTS, S, have been reported [6]. This encourages the pursuit for an effective vaccine for malaria. Vaccination effects in malaria control have been modeled mathematically [7,5,8]. Authors focus their attention on the vaccine's stage specificity, duration of effectiveness, responsiveness to natural boosting, vaccinated human proportions, pre-existing endemic conditions, and even economical effects.

The impact of other vector control interventions is studied in the literature. *P. falciparum* distribution in human populations, before the application of control measures, was described in [9]. Gu et al. [10] proposed an object-oriented design for individual-based modeling of malaria transmission by *Plasmodium falciparum*. The goal of the proposed model was to study the effect of infection control programs in interrupting malaria transmission, either by reducing human–vector contact rates or by implementing active case detection and drug treatment of infections. Authors concluded that malaria transmission was stable even in low transmission areas, where the human-biting rate was close to 0.5 bites per day. They have also verified that such kind of intervention in low transmission areas could lead to malaria extinction with a probability value close to 0.8. In medium and high transmission areas the probability of elimination was low, even with strong levels of intervention. The model was validated by epidemiological data collected at 30 sites along the coast of Kenya. In 2004, Depinay et al. [11] proposed a mathematical model for the dynamics of *Anopheles* population and their relation to the environment. It focused on five basic factors, temperature, moisture, nutrient competition, predation or death by disease, and dispersal. The authors concluded that the model provided a better understanding of the dynamics of *Anopheles* in specific local geographic environments. Gaudart et al. [12] used a SIRS-type model, for malaria transmission, where stochastic environmental factors were implemented. They found that the later influenced vector mortality and aggressiveness, and the length of the gonotrophic cycle. The model was adapted to Bancoumana's field study data. Authors concluded that models combining transmission patterns, predisposition factors and environmental variables might contribute for a better community risk evaluation. In 2011, White et al. [13] proposed a model to compare distinct interventions, such as indoor residual spraying, long-lasting insecticide treated nets, larvicides and pupicides, and their combinations, in malaria endemic settings in Sub-Saharan Africa. They argued that the selection of combinations of interventions, used in different stages in the vector's lifecycle, would strongly reduce *Anopheles gambiae* mosquitoes densities. Águas et al. [14] compared two mathematical models of transmission for *Plasmodium vivax* and *Plasmodium falciparum* parasites. Their work suggested that artemisinin-based combination therapy, combined with a hypnozoite killing drug, would eliminate both species. Nevertheless, *P. vivax*'s ability to relapse accelerated the acquisition of piecemeal clinical immunity. This parasite transmission persisted in areas of low mosquito abundance and was robust to drug administration initiatives due to relapse. Nevertheless, *P. vivax* was less lethal than *P. falciparum*.

Nowadays, though, malaria control depends mostly on personal protection, and treatment [15,16].

In this paper, we modify the mathematical model proposed by Chiyaka et al. [15], for the effects of control strategies on malaria transmission, to become a non integer order model. We aim to build a better approximation model to the real dynamics of malaria transmission among heterogeneous populations.

The framework of the paper is as follows. The model is described in Section 3. Simulation results are presented in Section 4. In Section 5, we infer conclusions from this work and list future research work.

2. Fractional calculus – brief summary

Researchers have been paying close attention to differentiation and integration to an arbitrary order, also known as Fractional Calculus (FC) [17–19]. Important applications of FC can be found in many areas of science, from electrochemistry [20], to physics [21,22], fluid mechanics [23], mechanical systems [24], other areas of engineering [25–28], and biology [29–31].

There are three important and well-studied definitions for FC. Namely, the Riemann–Liouville, the Grünwald–Letnikov, and the Caputo formula [19,17]. In this paper, we will consider the Grünwald–Letnikov derivative here, given by:

$$D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{k=0}^{\left\lfloor \frac{t-a}{h} \right\rfloor} (-1)^k \binom{\alpha}{k} f(t - kh) \quad (1)$$

where the function $\lfloor \cdot \rfloor$ means the integer part of the argument and h represents the time step increment.

Fractional derivatives have the unique property of capturing the history of the variable, that is, they have memory. This cannot be easily done by means of the integer order derivatives.

The numerical calculation of the fractional order derivative, by the Grünwald–Letnikov definition, can be based on the approximation of the time increment h through the sampling period T , and the series truncation at the r th term. This procedure is commonly known as Power Series Expansion (PSE), translating in the following equation in the z -domain:

$$Z\{D^\alpha x(t)\} \left[\frac{1}{T^\alpha} \sum_{k=0}^r \frac{(-1)^k \Gamma(\alpha + 1)}{k! \Gamma(\alpha - k + 1)} z^{-k} \right] X(z) \quad (2)$$

where $X(z) = Z\{x(t)\}$ and z and Z represent the z -transform variable and operator, respectively. In fact, expression (2) represents the Euler (or first backward difference) approximation in the $s \rightarrow z$ discretization scheme. Beside this formula are often adopted the Tustin and Al-Alaoui approximations [32]. The most often adopted generalization of the generalized derivative operator consists in $\alpha \in \mathbf{R}$.

3. The fractional model for malaria transmission

We consider a fractional version of the delay ordinary differential equations model proposed by Chiyaka et al. [15]. The model describes the dynamics of a population susceptible to infection by malaria, where personal protection and vaccination are implemented. The probability of transmission is reduced with treatment, since infectiousness of the individual, by decreasing the parasite load, is lower [15,8]. These strategies have the effect of reducing the number of infectives in a population.

3.1. Description of the model

We consider two populations: human and mosquitoes, that are divided into nine classes, six of them being human and the remaining three mosquito.

We denote by $N_h(t)$ the total number of individuals in the human population. It is given by $N_h(t) = S_h(t) + V_h(t) + E_h(t) + I_h(t) + Y_h(t) + T_h(t)$, where $\{S_h(t), V_h(t), E_h(t), I_h(t), Y_h(t), T_h(t)\}$ represent, respectively, {Susceptibles, Vaccinated, Exposed, Infectious, Infectious Vaccinated, Treated} humans.

The total number of individuals in the mosquito population, $N_m(t)$, is given by $N_m(t) = S_m(t) + E_m(t) + I_m(t)$, where $\{S_m(t), E_m(t), I_m(t)\}$ denote, respectively, {Susceptible, Exposed, Infectious} mosquitoes [15]. Fig. 1 shows the natural progression of the disease. The rate at which new individuals enter the human population by immigration or by birth is Λ_h . All humans die at a rate μ_h . The proportion of successful vaccinated humans is $p \in [0, 1)$, where $p\Lambda_h$ is the percentage of humans entering the $V_h(t)$ class, and $(1 - p)\Lambda_h$ is the proportion entering the susceptible class $S_h(t)$. The rate of infection of susceptible humans by the malaria parasite, $f_h(t)$, is given by:

$$f_h(t) = \beta_h c (1 - bz) \frac{I_m(t)}{N_h(t)} \quad (3)$$

where β_h is the probability that a susceptible human is infected after being bitten by an infectious mosquito and c is the rate of female mosquitoes' bites. Infected susceptible humans move to the exposed class $E_h(t)$. Individual protection is modeled by $(1 - b\zeta)$, where $0 < \zeta \leq 1$ measures the efficacy of adopted strategies for individual protection, and $0 < b \leq 1$ is the proportion of individuals in the community that use this protection strategy. Protection of successfully vaccinated humans may be only partial, and therefore they may develop disease at a rate $f_h(t)(1 - \gamma)$, where $0 \leq \gamma \leq 1$ is the efficacy of the vaccine pre-erythrocytic. Eventually, immunity induced by vaccination decreases to zero, and thus vaccinated individuals move to the susceptible class at a rate σ . Parameter κ is the rate at which infectious humans, I_h , are treated, and enter the treated class T_h . On the other hand, infectious humans may recover at a rate r_h and become susceptible, or even die from infection at a rate α_h . Analogous behavior is described for infectious vaccinated humans, Y_h . They are treated at a rate κ , recover at a rate $\theta_1 r_h$ or die from disease at a rate $(1 - \theta_2)\alpha_h$. Effects of the erythrocytic vaccine in increasing recovery and in reducing mortality due to disease are modeled by parameters $\theta_1 \geq 1$ and $0 < \theta_2 \leq 1$, respectively. Vaccinated humans V_h individuals do not recover. Treated humans, T_h , recover at a rate δr_h , where $\delta > 1$ models the efficacy of the drug in increasing recovery rate, or die at a rate $(1 - v)\alpha_h$, where $0 < v \leq 1$ determines the drug's efficacy as a factor in reducing infectious individuals' deaths caused by the disease.

The transmission dynamics of the mosquitoes population is described as follows. Susceptible mosquitoes are recruited at a constant rate Λ_m . Mosquitoes in every class are subjected to a natural death, which occurs at a rate μ_m . The rate $f_m(t)$ at which susceptible mosquitoes, S_m , get infected by the malaria parasite is given by:

$$f_m(t) = \beta_m c (1 - bz) \frac{I_h(t) + (1 - \epsilon)Y_h(t) + (1 - \eta)T_h(t)}{N_h(t)} \quad (4)$$

where β_m is the probability that a mosquito is infected after having bitted a susceptible human, carrying infectious gametophytes. Vaccine efficacy in blocking disease transmission is modeled by parameter $\epsilon \in [0, 1]$. The drug's effect in reducing malaria transmission from treated humans to mosquitoes, is modeled by parameter $\eta \in [0, 1]$. Susceptible infected mosquitoes move to the exposed class, and become infectious after a time period τ_m . There is an increase in the mosquitoes mortality rate, due to the presence of the parasite in their body at a rate α_m , which translates in a non recovering [15].

The system of delay differential equations for the proposed model is the following:

$$\begin{aligned} \frac{dN_h^{\alpha_1}(t)}{dt^{\alpha_1}} &= \Lambda_h - \xi_h(I_h(t) + (1 - \theta_2)Y_h(t) + (1 - v)T_h(t)) - \mu_h N_h(t) \\ \frac{dS_h^{\alpha_2}(t)}{dt^{\alpha_2}} &= (1 - p)\Lambda_{hj} - f_h(t)S_h(t) + r_h(I_h(t) + \theta_1 Y_h(t) + \delta T_h(t)) + \sigma V_h(t) - \mu_h S_h(t) \end{aligned}$$

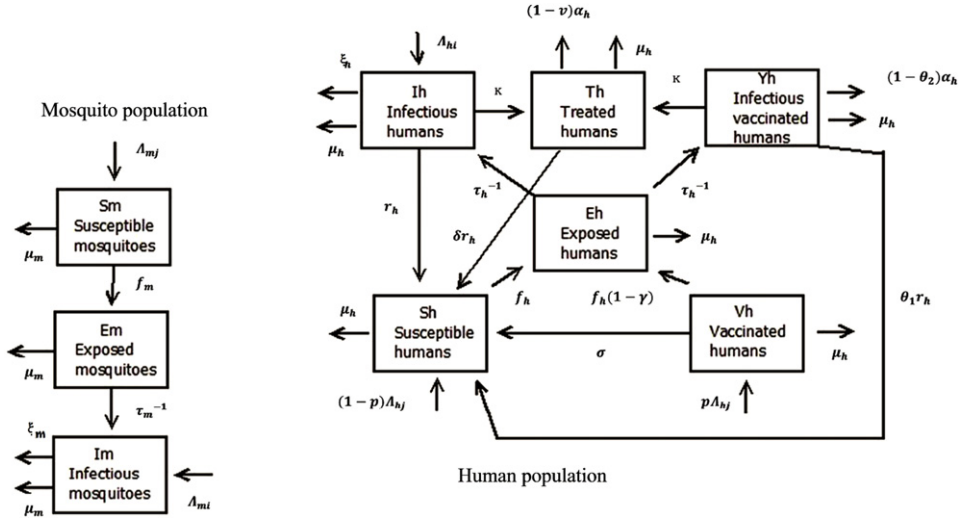


Fig. 1. Illustration of the transmission dynamics of malaria.

$$\begin{aligned}
 \frac{dV_h^{\alpha_3}(t)}{dt^{\alpha_3}} &= p\Lambda_{hj} - f_h(t)(1-\gamma)V_h(t) - (\sigma + \mu_h)V_h(t) \\
 \frac{dI_h^{\alpha_4}(t)}{dt^{\alpha_4}} &= f_h(t - \tau_h)S_h(t - \tau_h)e^{-\mu_h\tau_h} - (\kappa + r_h + \xi_h + \mu_h)I_h(t) \\
 \frac{dY_h^{\alpha_5}(t)}{dt^{\alpha_5}} &= f_h(t - \tau_h)(1-\gamma)V_h(t - \tau_h)e^{-\mu_h\tau_h} - (\kappa + \theta_1 r_h + (1-\theta_2)\xi_h + \mu_h)Y_h(t) \\
 \frac{dT_h^{\alpha_6}(t)}{dt^{\alpha_6}} &= \kappa(I_h(t) + Y_h(t)) - (\delta r_h + (1-v)\xi_h + \mu_h)T_h(t) \\
 \frac{dN_m^{\alpha_7}(t)}{dt^{\alpha_7}} &= \Lambda_m - \xi_m I_m(t) - \mu_m N_m(t) \\
 \frac{dS_m^{\alpha_8}(t)}{dt^{\alpha_8}} &= \Lambda_m - f_m(t)S_m(t) - \mu_m S_m(t) \\
 \frac{dI_m^{\alpha_9}(t)}{dt^{\alpha_9}} &= f_m(t - \tau_m)S_m(t - \tau_m)e^{-\mu_m\tau_m} - (\mu_m + \xi_m)I_m(t) + \Lambda_m
 \end{aligned} \tag{5}$$

where $\alpha = (\alpha_1, \dots, \alpha_9)$, $\alpha_i \in [0, 1]$, $i = 1, \dots, 9$, is the order of the fractional derivative.

4. Numerical simulations

In this section, we present numerical simulations of model (5). We study the dynamical behavior of the model for variation of the non integer order derivative α , where $\alpha = \alpha_1 = \alpha_2 = \dots = \alpha_8$, and variation of the parameter b , the proportion of individuals in the community that use protection.

The parameter values used in the simulations can be found in Table 1 and the initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$. For the sake of clarity and due to the complexity of its interpretation in the fractional order case, the physical measurement units are not presented.

For the numerical implementation of the fractional derivatives a series expansion based on the Grünwald–Letnikov definition was adopted. Several numerical experiments demonstrated that the large period of integration and the delay lead to precision problems in the final results. Therefore, was considered a small time increment step and a large number of terms in the truncated series, namely $dt = 10^{-5}$ days and $rmax = 10^4$ terms. In Figs. 2–7 we simulate, approximately, 300 d. From observation of the figures we conclude that as b is increased from 0 to 1 there is a corresponding decrease in the number of infectious humans and infectious mosquitoes, for all values of α . This means that this result is invariant for variation of fractional derivative, in the values tested. The results are in accordance with the results obtained in the paper [15] for $\alpha = 1.0$.

5. Conclusions

In this paper, we study numerically a fractional model for malaria transmission under control strategies. We consider the integer order model proposed by Chiyaka et al. in [15] and modify it to become a fractional order model. In the numerical

Table 1

Parameters used in the numerical simulations of model (5).

Parameter	Value
Λ_h	0.05
p	0.8
σ	0.009
β_h	0.5
c	0.5
b	0.8
ζ	0.6
r_h	0.005
ξ_h	0.0004
μ_h	0.0000391
γ	0.64
ϵ	0.86
τ_h	14
θ_1	4.1
θ_2	0.06
κ	0.2
δ	8.04
v	0.02
η	0.86
Λ_m	4.0
β_m	0.83
μ_m	0.04
τ_m	12
ξ_m	0.01

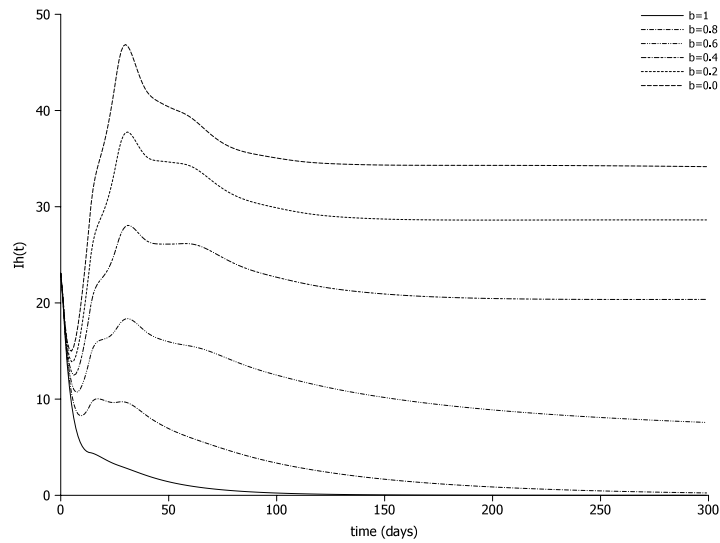


Fig. 2. Evolution of infectious humans in the absence of treatment and vaccination, for varying b from 0 to 1 in steps of 0.2. Parameter values are those of Table 1 and $z = 0.9$. Initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$, for a value of the non integer order derivative of $\alpha = 0.4$.

simulations, we vary the values of the fractional derivative and of the parameter that models personal protection, b . From observation of the graphs, we conclude that as b is increased from 0 to 1 there is a corresponding decrease in the number of infectious humans and infectious mosquitoes, for all values of α . This means that this result is invariant for variation of fractional derivative, in the values tested. These results are in agreement with those obtained in [15] for $\alpha = 1.0$ and suggest that our fractional model is epidemiologically well-posed.

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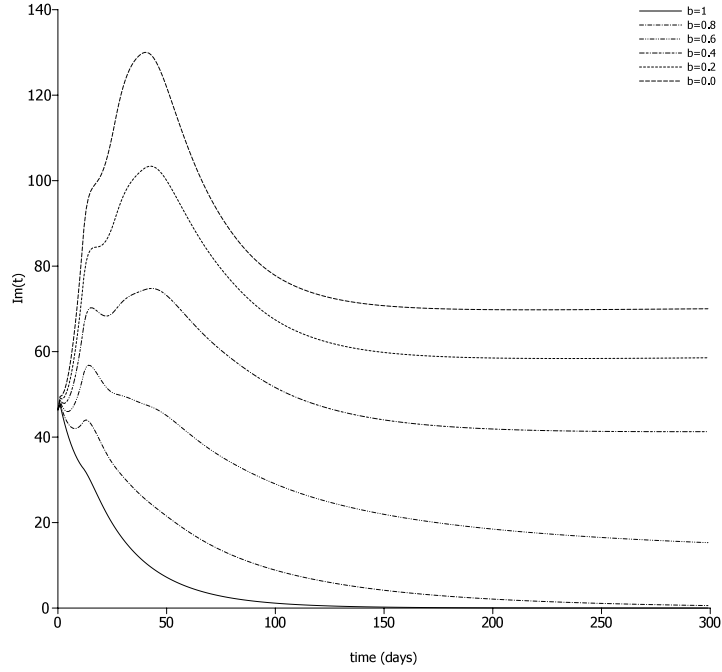


Fig. 3. Evolution of infectious mosquitoes in the absence of treatment and vaccination, for varying b from 0 to 1 in steps of 0.2. Parameter values are those of Table 1 and $z = 0.9$. Initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$, for a value of the non integer order derivative of $\alpha = 0.4$.

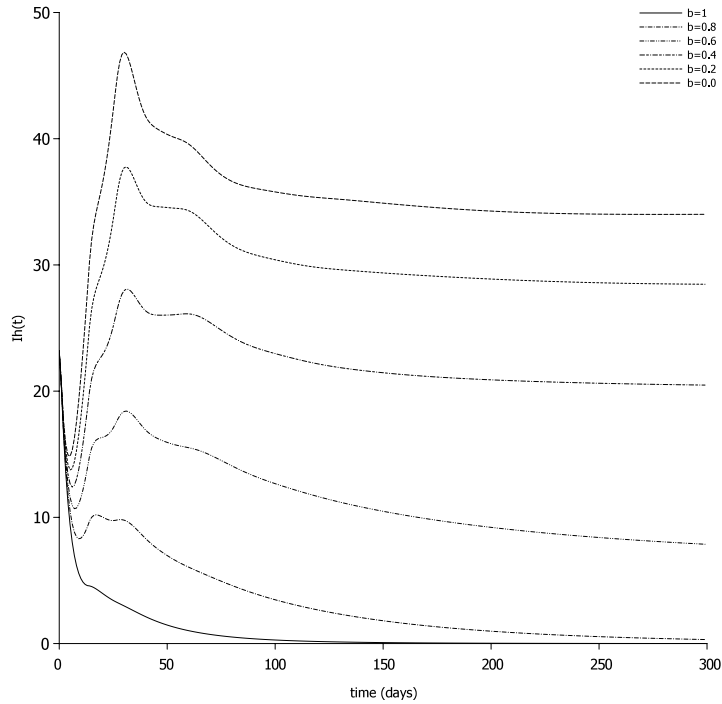


Fig. 4. Evolution of infectious humans in the absence of treatment and vaccination, for varying b from 0 to 1 in steps of 0.2. Parameter values are those of Table 1 and $z = 0.9$. Initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$, for a value of the non integer order derivative of $\alpha = 0.6$.

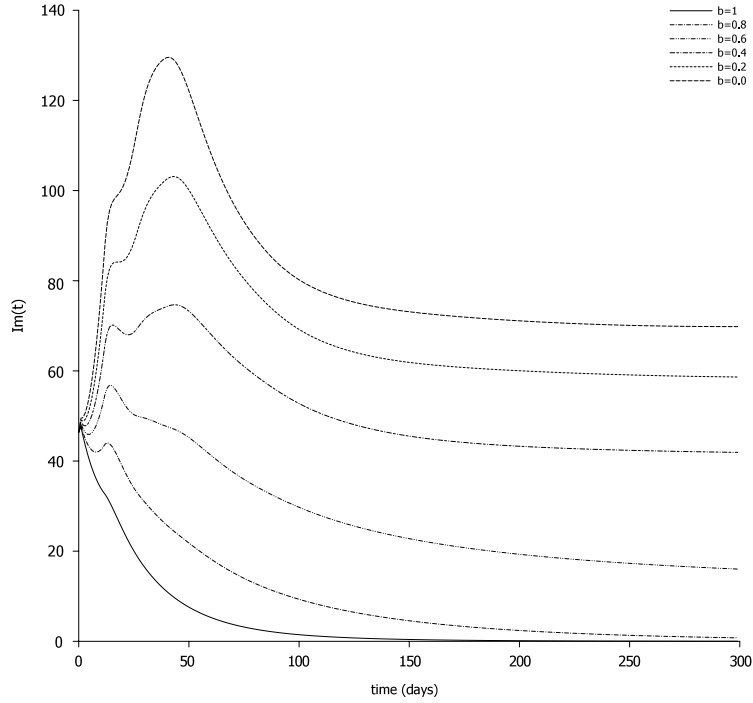


Fig. 5. Evolution of infectious mosquitoes in the absence of treatment and vaccination, for varying b from 0 to 1 in steps of 0.2. Parameter values are those of Table 1 and $z = 0.9$. Initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$, for a value of the non integer order derivative of $\alpha = 0.6$.

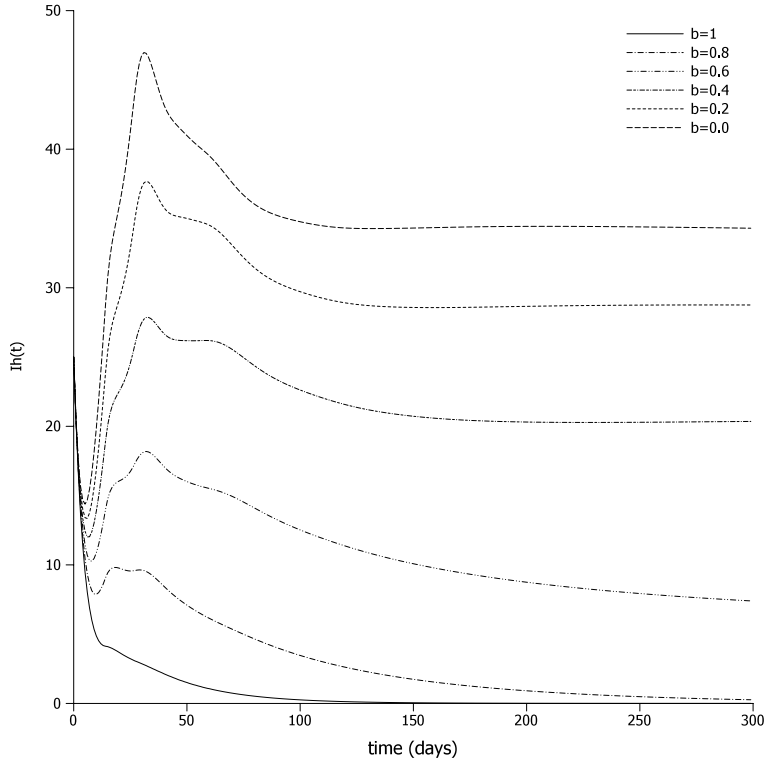


Fig. 6. Evolution of infectious humans in the absence of treatment and vaccination, for varying b from 0 to 1 in steps of 0.2. Parameter values are those of Table 1 and $z = 0.9$. Initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$, for a value of the non integer order derivative of $\alpha = 1.0$.

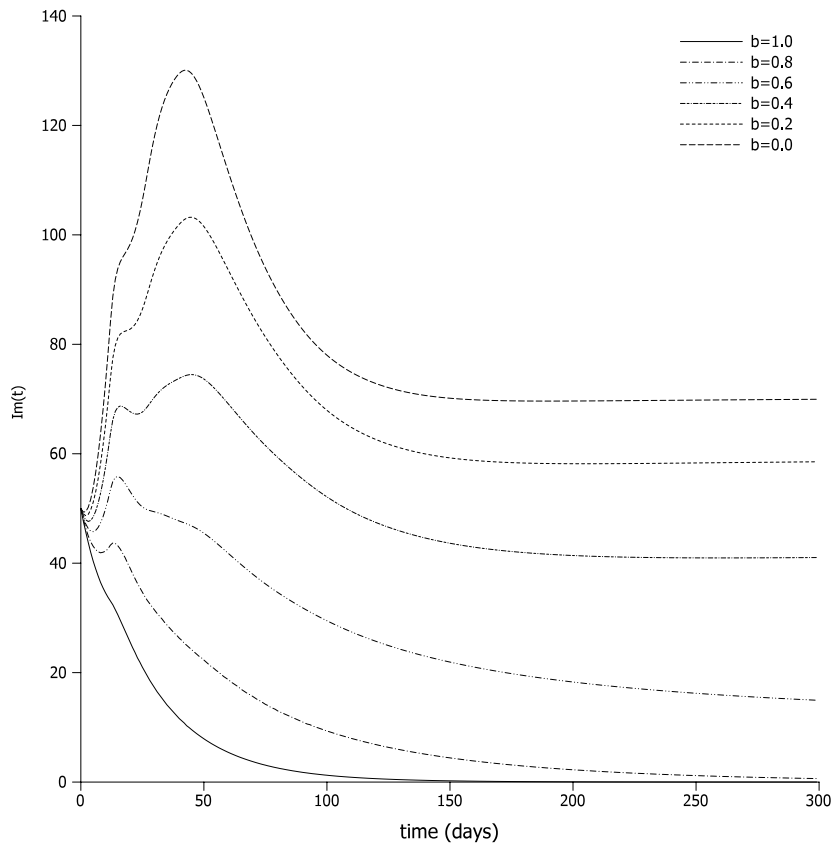


Fig. 7. Evolution of infectious mosquitoes in the absence of treatment and vaccination, for varying b from 0 to 1 in steps of 0.2. Parameter values are those of Table 1 and $z = 0.9$. Initial conditions are $N_h(t) = 450$, $S_h(t) = 300$, $V_h(t) = 100$, $I_h(t) = 25$, $Y_h(t) = 5$, $T_h(t) = 0$, $N_m(t) = 480$, $S_m(t) = 430$, $I_m(t) = 20$, for a value of the non integer order derivative of $\alpha = 1.0$.

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