



Nonlinear Dynamics

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The modeling and control of nonlinear dynamic systems is challenging in mathematics and engineering. Despite much investigation carried out so far, many nonlinear and complex phenomena are not fully understood yet, due to their considerable randomness and a diversity of reasons underlying the energy dissipation involving the dynamic effects. Indeed, present knowledge and scientific tools are still far from capturing the overall richness of the systems dynamics. The Special Issue “Nonlinear Dynamics” focuses on original and new research results on modeling and control of dynamical systems in science and engineering. It includes 12 manuscripts addressing novel issues and specific topics that illustrate the richness and challenging nature of nonlinear dynamics. In the follow-up, the selected manuscripts are presented in alphabetic order of their titles.

In the paper “A New Generalized Taylor-Like Explicit Method for Stiff Ordinary Differential Equations”, El-Zahar et al. [1] present a new generalized Taylor-like explicit method for stiff ordinary differential equations. The method is shown to have an arbitrary high order of convergence and L -stability. Additionally, several integration schemes are proven to be special cases of the new generalized form presented. Finally, the authors demonstrate that the new method overcomes several shortcomings revealed by the classical Taylor-like schemes both in their component and vector forms.

The manuscript “Computer Analysis of Human Belligerency” by Machado, Lopes and Mata [2] adopts hierarchical clustering techniques and multidimensional scaling as efficient instruments for visualizing and describing military conflicts. Different metrics are used to assess the events, based on suitable features, and entropy is adopted for measuring war complexity over time. The authors verify a growing number of events and an explosion in their characteristics. The results obtained have similarities to those exhibited by systems with increasing volatility or evolving toward chaotic-like behavior. It is conjectured whether such dynamics follows the second law of thermodynamics, since the adopted techniques reflect a system expanding entropy.

In “Ethanol Prices and Agricultural Commodities: An Investigation of Their Relationship” [3], the authors David, Inácio Jr., and Machado propose the Bai–Perron test of breakpoints, the Johansen cointegration test, and the vector error correction model exploited by the orthogonal impulse response and the forecast error variance decomposition, to analyze the price transmission among ethanol and agricultural commodities data series in Brazil. It is shown that there is a strong price transmission from the ethanol toward the agricultural commodities, but not in the opposite direction.

In their work “LMI-Observer-Based Stabilizer for Chaotic Systems in the Existence of a Nonlinear Function and Perturbation” [4], Karami et al. present an observer-based state feedback stabilizer for a class of chaotic systems with external perturbations and Lipschitz nonlinearities. The stabilizer and observer parameters are obtained from linear matrix inequalities. Moreover, design techniques are investigated for nonlinear systems with an output nonlinear function. Numerical examples illustrate the convergence to zero of the estimation errors, the Lyapunov stability of the closed-loop system, and the cancellation of the effects of perturbation and nonlinearities.



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The paper “New Irregular Solutions in the Spatially Distributed Fermi–Pasta–Ulam Problem” [5] by Kashchenko and Tolbey presents the construction of families of special nonlinear systems of the Schrödinger-type quasinormal forms, whose nonlocal dynamics determines the local behavior of solutions to the spatially distributed Fermi–Pasta–Ulam equation. Irregular solutions are studied that contain components rapidly oscillating in the spatial variable, with different asymptotically large modes.

In the work “Nonlinear Dynamics and Control of a Cube Robot” [6], Liao et al. model and design a cube robot capable of self-bouncing and self-balancing. Firstly, the dynamic model of the robot is derived by using the conservation of the angular momentum and the torque equilibrium theory. Secondly, the controllability of the robot is analyzed and its angle is determined using an attitude and heading reference system. Thirdly, a controller for the robot is proposed. Finally, experimental results are presented to demonstrate the effectiveness of the robot and its robustness to external disturbances.

In “On Complete Monotonicity of Solution to the Fractional Relaxation Equation with the n th Level Fractional Derivative” [7], Luchko deduces explicit formulas for the projector of the n th level fractional derivative and for its Laplace transform. Then, he discusses the fractional relaxation equation with the n th level fractional derivative. The author shows that, under some conditions, the solutions to the initial-value problems of the equation are completely monotone functions that can be represented by linear combinations of Mittag–Leffler functions with power law weights.

The manuscript “On the Poisson Stability to Study a Fourth-Order Dynamical System with Quadratic Nonlinearities” [8], authored by Pchelintsev, discusses the search procedure for Poincaré recurrences to classify solutions on an attractor of a fourth-order nonlinear dynamical system. For the resulting limiting solution, the Lyapunov exponents are calculated using the modified Benettin’s algorithm, to study the stability of the regime found, and to confirm the type of attractor.

In “Revisiting the 1D and 2D Laplace Transforms” [9], Ortigueira and Machado review the unilateral and bilateral one- and two-dimensional Laplace transforms. These transforms are compared in the one-dimensional case, leading to the formulation of the initial-condition theorem. The problem is solved in the one- and two-dimensional cases with the bilateral Laplace transform. The case of fractional-order systems is also addressed. General two-dimensional linear systems are introduced and the corresponding transfer function is defined.

In their paper “Schistosomiasis Model Incorporating Snail Predator as Biological Control Agent” [10], Nur et al. propose a schistosomiasis model that incorporates a snail predator as a biological control agent. The authors study the condition for the existence of equilibrium points and show that the solutions of the model are non-negative and bounded. The basic reproduction number is determined when the predator goes to extinction and when it survives. The results suggest that a snail predator agent can reduce schistosomiasis prevalence. Moreover, the snail-predator birth rate plays an essential role in controlling schistosomiasis spread.

In the paper “Stability Switches and Double Hopf Bifurcation Analysis on Two-Degree-of-Freedom Coupled Delay van der Pol Oscillator” [11], Chen and Qian study two-degrees-of-freedom coupled van der Pol oscillators with time delay feedback using the normal form and central manifold theories. The results show that change in time delay causes stability switching of the system, and a greater time delay translates into a more complex stability switching.

In “Unpredictable Solutions of Linear Impulsive Systems” [12], Akhmet et al. address a new type of oscillations of discontinuous unpredictable solutions for linear impulsive nonhomogeneous systems. A definition of a piecewise continuous unpredictable function is provided, and theoretical results on the existence, uniqueness, and stability of discontinuous unpredictable solutions for linear impulsive differential equations are given. The authors use randomly determined unpredictable sequences, where the construction of a discontinuous unpredictable function is based on an unpredictable sequence determined

by a discrete random process, and the set of discontinuity moments is obtained by the logistic map. Numerical examples illustrate the theoretical findings.

To sum up, the guest editors hope that the selected papers will help scholars and researchers to push forward the progress in Nonlinear Dynamics, namely the modeling and control of nonlinear systems.

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