

Symmetry in Complex Systems

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Complex systems with symmetry arise in many fields, at various length scales, including financial markets, social, transportation, telecommunication and power grid networks, world and country economies, ecosystems, molecular dynamics, immunology, living organisms, computational systems, and celestial and continuum mechanics. The emergence of new order and structure in complex systems means symmetry breaking and transition from unstable to stable states. Modeling complexity attracted many researchers from different areas, dealing both with theoretical concepts and practical applications. This Special Issue seeks to fill the gap between the theory of symmetry-based dynamics and its application to model and analyze complex systems. This Special Issue focuses on the synergies between the theory of symmetry-based dynamics and its application to model and analyze complex systems. It includes 7 manuscripts addressing novel issues and specific topics that illustrate symmetry in complex systems. In the follow-up the selected manuscripts are presented in alphabetic order.

The manuscript “A Transmission Prediction Neighbor Mechanism Based on a Mixed Probability Model in an Opportunistic Complex Social Network” [1], by Genghua Yu, Zhigang Chen, Jia Wu and Jian Wu, proposes a routing decision method based on an improved probability model combined with a quantitative social relationship value and cooperative value to filter neighbor nodes. The algorithm combines multiple feature information between nodes and uses this feature information to quantify social relationship values and partnership values. Then, the prediction matrix is obtained by matrix decomposition and gradient descent, and the relay nodes are filtered according to the predicted probability values.

In the paper “Derivative Free Fourth Order Solvers of Equations with Applications in Applied Disciplines” [2], Ramandeep Behl, Ioannis K. Argyros, Fouad Othman Mallawi and J. A. Tenreiro Machado address efficient equation solvers for real- and complex-valued functions. The work extends earlier schemes and studies the computable radii of convergence and error bounds based on the Lipschitz constants. Furthermore, the range of starting points is explored to know how close the initial guess should be considered for assuring convergence.

In the work “Extending the Adapted PageRank Algorithm Centrality to Multiplex Networks with Data Using the PageRank Two-Layer Approach” [3], Taras Agryzkov, Manuel Curado, Francisco Pedroche, Leandro Tortosa and José F. Vicent propose a measure of centrality for biplex networks based on the adapted PageRank algorithm centrality for spatial networks with data. The scheme is implemented following the two-layers approach for PageRank model. The new measure of centrality can determine the importance of the nodes of a network and work with several data sets associated with the nodes themselves, without any connection or relationship between them.

The manuscript “Mei Symmetry and Invariants of Quasi-Fractional Dynamical Systems with Non-Standard Lagrangians” [4], by Yi Zhang and Xue-Ping Wang deals with quasi-fractional dynamical

systems from exponential non-standard Lagrangians and power-law non-standard Lagrangians. Firstly, the definition, criterion, and corresponding new conserved quantity of Mei symmetry in this system are presented and studied. Secondly, considering that a small disturbance is applied on the system, the differential equations of the disturbed motion are established, the definition of Mei symmetry and corresponding criterion are given, and the new adiabatic invariants led by Mei symmetry are proposed and proved.

The paper “Multi-Agent Reinforcement Learning Using Linear Fuzzy Model Applied to Cooperative Mobile Robots” [5], by David Luviano-Cruz, Francesco Garcia-Luna, Luis Pérez-Domínguez and S. K. Gadi, presents a joint Q -function linearly fuzzified for a multi-agent system continuous state space, which overcomes the dimensionality problem. A proof for the convergence and existence of the solution proposed by the algorithm presented.

In the research “Time-Fractional Heat Conduction in a Plane with Two External Half-Infinite Line Slits under Heat Flux Loading” [6], Yuriy Povstenko and Tamara Kyrylych solve the time-fractional heat conduction equation with the Caputo derivative for an infinite plane with two external half-infinite slits with the prescribed heat flux across their surfaces. The integral transform technique is used. The solution is obtained in the form of integrals with the integrand being the Mittag-Leffler function.

The paper “Time-Fractional Heat Conduction in Two Joint Half-Planes” [7], by Yuriy Povstenko and Joanna Klekot, address the heat conduction equations with Caputo fractional derivative in two joint half-planes under the conditions of perfect thermal contact. The fundamental solution to the Cauchy problem as well as the fundamental solution to the source problem are examined. The Fourier and Laplace transforms are employed. The Fourier transforms are inverted analytically, whereas the Laplace transform is inverted numerically using the Gaver-Stehfest method.

The guest editors believe that the selected high-quality papers will help scholars and researchers to push forward the progress in the emerging area of symmetry in complex systems.

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