

Paper:

# Fractional-Order Position/Force Robot Control

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This paper studies fractional-order (*FO*) position/force algorithms in the hybrid control (*HC*) and the cascade control (*CC*) of manipulators. The system performance is analyzed, in the time and frequency domains, and the effects of joint backlash and flexibility are investigated. The results show that the *FO* algorithms and the *CC* architecture have superior robustness and stability.

**Keywords:** robotics, fractional calculus, control, force, backlash

## 1. Introduction

In the early eighties Raibert and Craig [1] introduced the concept of force control based on the hybrid algorithm. Since then, several researchers [2, 3] developed these ideas and proposed new algorithms such as the impedance controller. Problems with position/force control are further investigated in [3], while more recent studies of this algorithm can be found in [4–7].

There are two basic methods for force control, namely the hybrid position/force and the impedance schemes. The first algorithm [1] separates the task into two orthogonal subspaces corresponding to the force and the position subspaces. Once established the subspace decomposition two independent controllers are designed. Alternatively, with the second algorithm [2], by a proper choice of the arm impedance, the interaction forces can be accommodated to obtain an adequate response.

This paper studies the position/force control of robot manipulators, required in processes that involve contact between the gripper and the environment, using fractional-order (*FO*) algorithms [11]. The application of the theory of fractional calculus is still in a research stage, but the recent progress in this area reveals promising aspects for future developments [6, 9].

In this line of thought the article is organized as follows. Sections two and three introduce the hybrid (*HC*)

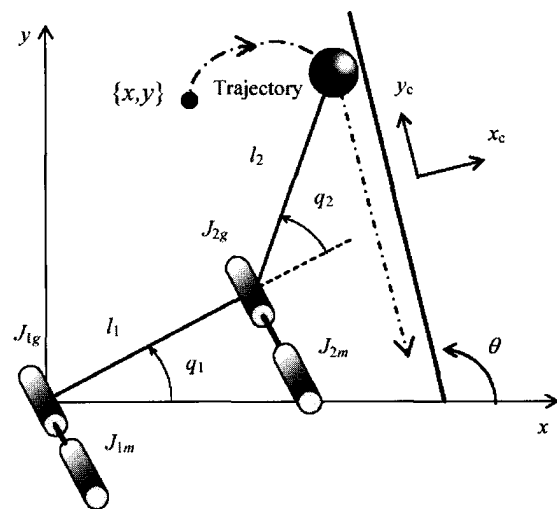


Fig. 1. The 2R robot and the constraint surface.

and the cascade (*CC*) controllers and the fundamentals of the *FO* algorithms, respectively. Section four analyses several experiments for the performance evaluation of two strategies, namely the *FO* and the *PID* controllers, for robots having several joint dynamical phenomena. Finally, section five outlines the main conclusions.

## 2. The Hybrid Controller

The dynamical equation of a  $n$  dof robot is:

$$\tau = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) - \mathbf{J}^T(\mathbf{q})\mathbf{F} \quad \dots \quad (1)$$

where  $\tau$  is the  $n \times 1$  vector of actuator torques,  $\mathbf{q}$  is the  $n \times 1$  vector of joint coordinates,  $\mathbf{H}(\mathbf{q})$  is the  $n \times n$  inertia matrix,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is the  $n \times 1$  vector of centrifugal/Coriolis terms and  $\mathbf{G}(\mathbf{q})$  is the  $n \times 1$  vector of gravitational effects. The  $n \times m$  matrix  $\mathbf{J}^T(\mathbf{q})$  is the transpose of the Jacobian of the robot and  $\mathbf{F}$  is the  $m \times 1$  vector of the force that the ( $m$ -dimensional) environment exerts in the gripper.

In this study we adopt the 2R robot (Fig.1) with dynamics:

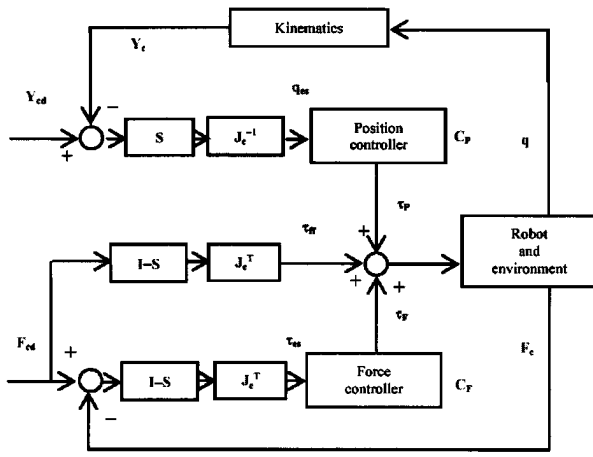


Fig. 2. The position/force hybrid controller.

$$H(q) = \begin{bmatrix} (m_1 + m_2)r_1^2 + m_2r_2^2 & m_2r_2^2 + m_2r_1r_2C_2 \\ +2m_2r_1r_2C_2 + J_{1m} + J_{1g} & m_2r_1r_2C_2 \\ m_2r_2^2 + m_2r_1r_2C_2 & m_2r_2^2 + J_{2m} + J_{2g} \end{bmatrix} \quad (2a)$$

$$C(q, \dot{q}) = \begin{bmatrix} -m_2r_1r_2S_2\dot{q}_2^2 - 2m_2r_1r_2S_2\dot{q}_1\dot{q}_2 \\ m_2r_1r_2S_2\dot{q}_1^2 \end{bmatrix} \quad (2b)$$

$$G(q) = \begin{bmatrix} g(m_1r_1C_1 + m_2r_1C_1 + m_2r_2C_{12}) \\ gm_2r_2C_{12} \end{bmatrix} \quad (2c)$$

$$J^T(q) = \begin{bmatrix} -r_1S_{11} - r_2S_{12} & r_1C_{11} + r_2C_{12} \\ -r_2S_{12} & r_2C_{12} \end{bmatrix} \quad (2d)$$

where  $C_{ij} = \cos(q_i + q_j)$  and  $S_{ij} = \sin(q_i + q_j)$ .

The numerical values adopted for the robot are  $m_1 = 0.5\text{kg}$ ,  $m_2 = 6.25\text{kg}$ ,  $r_1 = 1.0\text{m}$ ,  $r_2 = 0.8\text{m}$ ,  $J_{1m} = J_{2m} = 1.0\text{kgm}^2$  and  $J_{1g} = J_{2g} = 4.0\text{kgm}^2$  [6].

The constraint plane is determined by the angle  $\theta$  (Fig.1) and the contact displacement  $x_c$  of the robot gripper with the constraint surface is modeled through a linear system with a mass  $M$ , a damping  $B$  and a stiffness  $K$  with dynamics:

$$F_c = M\ddot{x}_c + B\dot{x}_c + Kx_c \quad (3)$$

The first control architecture consists on the *HC* algorithm (Fig.2). The diagonal  $n \times n$  selection matrix  $S$  has elements equal to one (zero) in the position (force) controlled directions and  $I$  is the  $n \times n$  identity matrix. In this paper the  $y_c(x_c)$  cartesian coordinate is position (force) controlled, yielding:

$$S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, J_c(q) = \begin{bmatrix} -r_1C_{\theta 11} - r_2C_{\theta 12} & -r_2C_{\theta 12} \\ r_1S_{\theta 11} + r_2S_{\theta 12} & r_2S_{\theta 12} \end{bmatrix} \quad (4)$$

where  $C_{\theta ij} = \cos(\theta - q_i - q_j)$  and  $S_{\theta ij} = \sin(\theta - q_i - q_j)$ .

The *CC* architecture (Fig.3) is inspired on the impedance and compliance schemes. Therefore, we es-

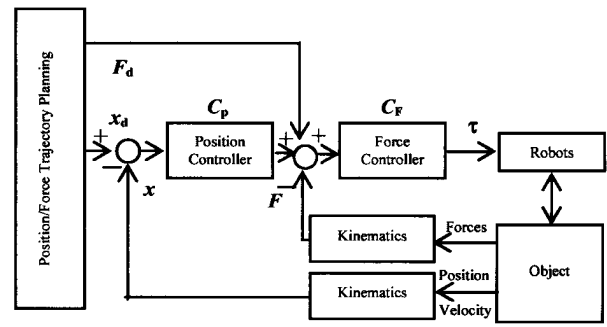


Fig. 3. The position/force cascade controller.

tablish a cascade of force and position algorithms as internal and external feedback loops, respectively, where  $x_d$  and  $F_d$  are the payload desired position coordinates and contact forces.

### 3. Fractional Order Algorithms

In this section we present the *FO* algorithms inserted at the position and force control loops.

The mathematical definition of a derivative of fractional order  $\alpha$  has been the subject of several different approaches. For example, we can mention the Laplace and the Grünwald-Letnikov definitions:

$$D^\alpha[x(t)] = L^{-1}\{s^\alpha X(s)\} \quad (5a)$$

$$D^\alpha[x(t)] = \lim_{h \rightarrow 0} \left[ \frac{1}{h^\alpha} \sum_{k=1}^{\infty} \frac{(-1)^k \Gamma(\alpha + 1)}{\Gamma(k + 1) \Gamma(\alpha - k + 1)} x(t - kh) \right] \quad (5b)$$

where  $\Gamma$  is the gamma function and  $h$  is the time increment. In our case, for implementing *FO* algorithms of the type  $C(s) = K_0 + Ks^\alpha$ ,  $-1 < \alpha < 1$ , we adopt a 4<sup>th</sup>-order discrete-time Pade approximation ( $a_i, b_i, c_i, d_i \in \mathbb{R}, k = 4$ ):

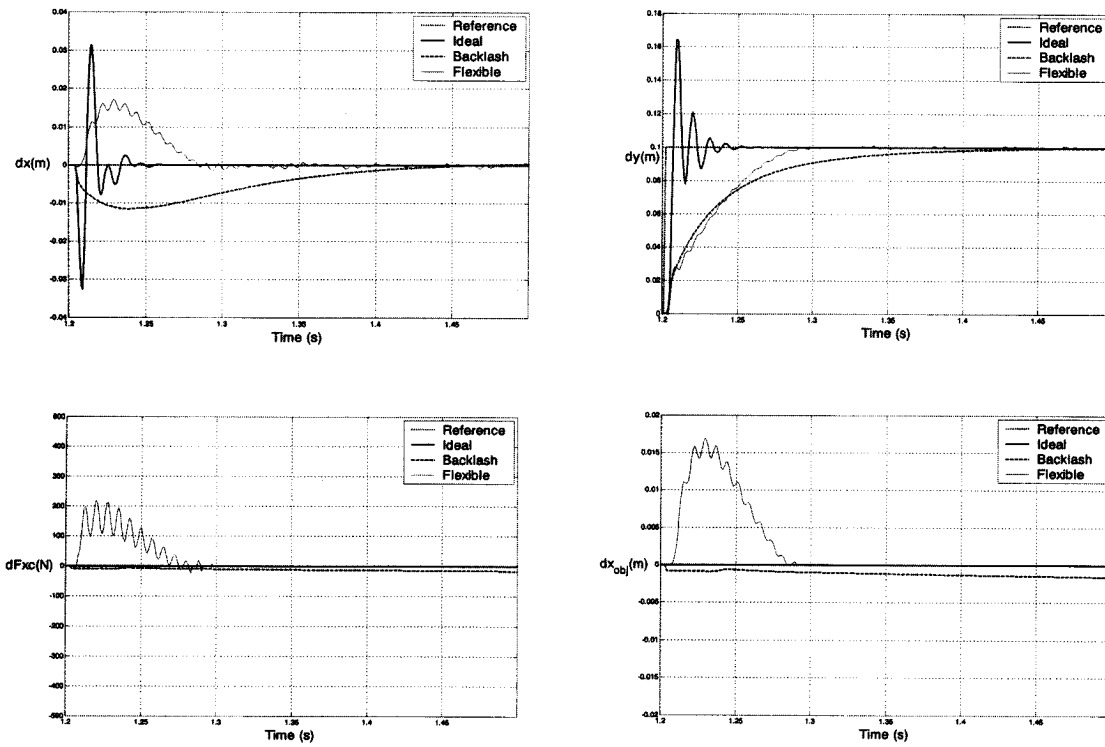
$$C_P(z) \approx K_P \frac{a_0z^k + a_1z^{k-1} + \dots + a_k}{b_0z^k + b_1z^{k-1} + \dots + b_k} \quad (6a)$$

$$C_F(z) \approx K_F \frac{c_0z^k + c_1z^{k-1} + \dots + c_k}{d_0z^k + d_1z^{k-1} + \dots + d_k} \quad (6b)$$

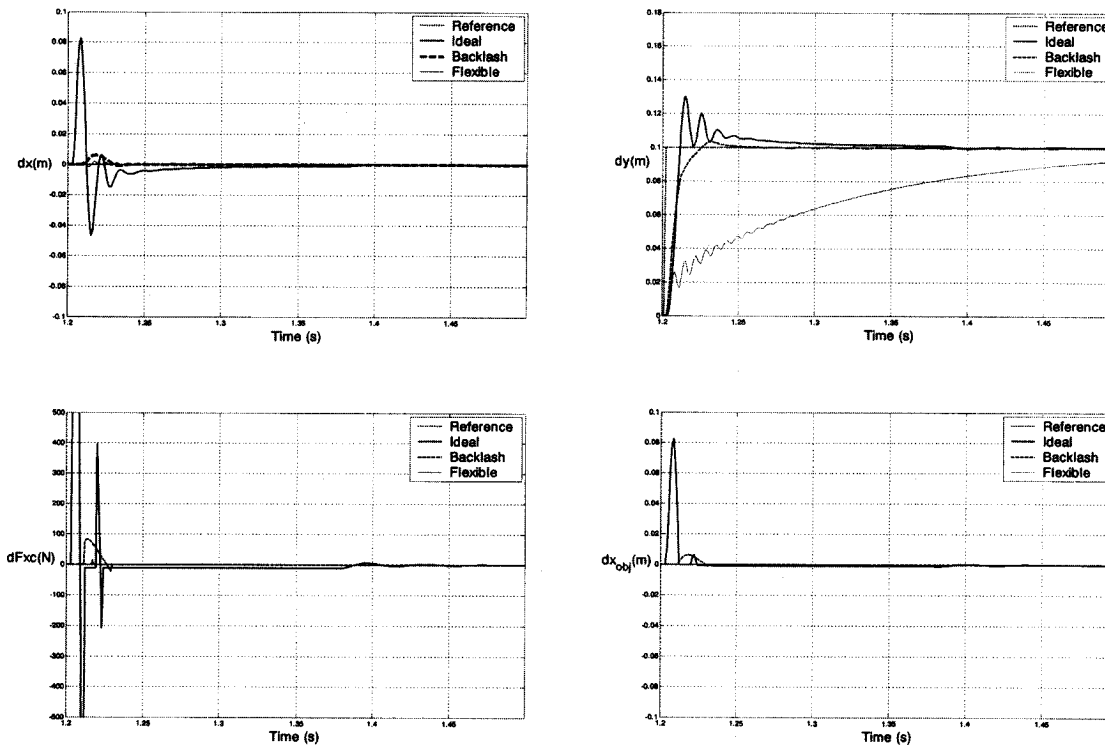
where  $K_P$  and  $K_F$  are the position and force loop gains, respectively.

### 4. Controller Performances

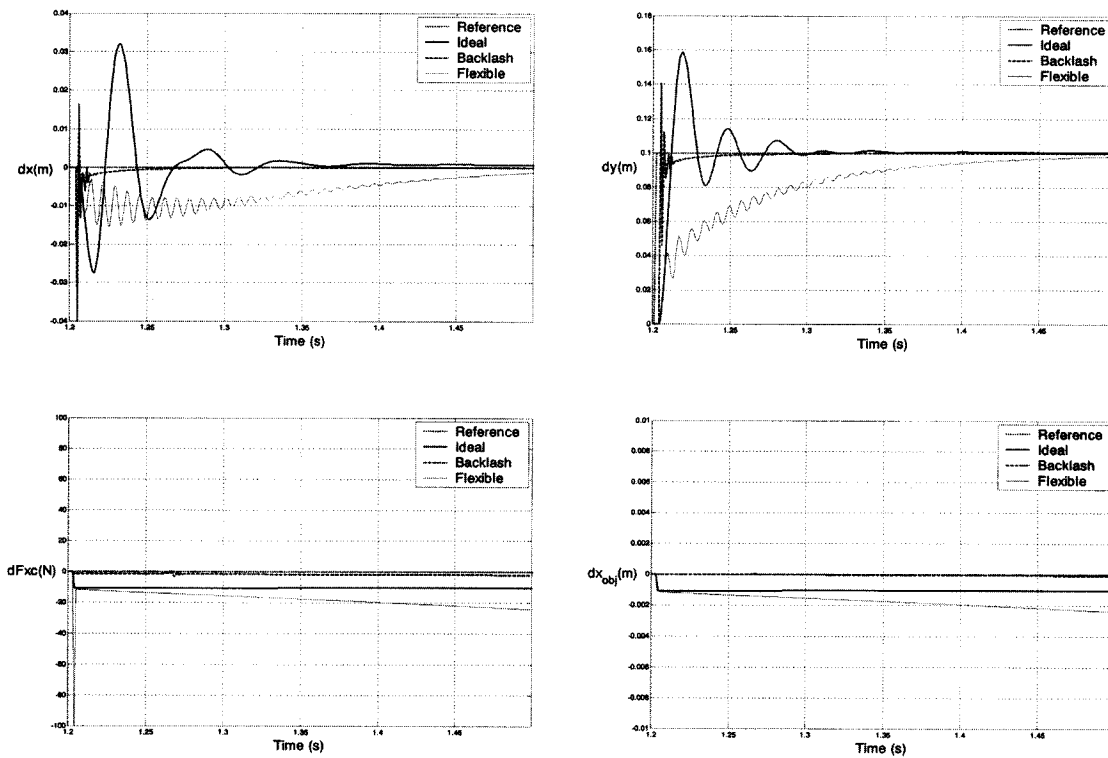
This section analyzes the system performance both for robots with ideal transmissions and robots with dynamic phenomena at the joints, such as backlash and flexibility. Furthermore, we compare the response of classical *PID* and *FO* algorithms. For the integer-order case we adopt a *PD*:  $C_P(s) = K_p + K_d s$  and a *PI*:  $C_F(s) = K_p + K_i s^{-1}$  controllers, in the position and force loops [1–3].



**Fig. 4.** Time response for the 2R ideal robot and robots with joints having backlash and flexibility under the action of the  $PD-PI$  algorithm for the  $HC$  and  $\delta y_d = 10^{-3} \text{ m}$ .



**Fig. 5.** Time response for the 2R ideal robot and robots with joints having backlash and flexibility under the action of the  $FO$  algorithm for the  $HC$  and  $\delta y_d = 10^{-3} \text{ m}$ .



**Fig. 6.** Time response for the 2R ideal robot and robots with joints having backlash and flexibility under the action of the PD – PI algorithm for the CC and  $\delta y_d = 10^{-3}$  m.

Due to the highly coupled and nonlinear nature of the system there is no systematic analytical procedure for adjusting the control parameters. Therefore, both algorithms were tuned by trial and error, based on the human experience and intuition, having in mind getting a good performance in the two cases. The alternative of computational-based schemes such as, for example, genetic algorithms, lead to similar problems of guarantying that optimality of the proposed tuning. Consequently, it remains an open problem the design of more systematic design methods.

The resulting parameters for the HC were  $FO : \{K_P, K_D, \alpha_P\} \equiv \{1.12 \times 10^3, 1.5 \times 10^{-3}, 0.5\}$ ,  $\{K_F, K_I, g\alpha_F\} \equiv \{56.23, 1.83 \times 10^{-3}, -0.2\}$  and  $PD/PI : \{K_p, K_d\} \equiv \{10^4, 10^3\}$ ,  $\{K_p, K_i\} \equiv \{10^3, 10^2\}$  for the position and force loops, respectively. The parameters for the CC are  $FO : \{K_P, K_D, \alpha_P\} \equiv \{0.1259, 0.15 \times 10^{-3}, 0.5\}$ ,  $\{K_F, K_I, \alpha_F\} \equiv \{10.59, 1.83 \times 10^{-3}, -0.2\}$  and  $PD/PI : \{K_p, K_d\} \equiv \{10^4, 10^3\}$ ,  $\{K_p, K_i\} \equiv \{10^2, 10\}$  for the position and force loops, respectively. Moreover, it is adopted a loop trajectory starting at the operating point  $\{x, y\} \equiv \{1, 1\}$  in the open space, having contact after one second with a constraint surface with parameters  $\{\theta, M, B, K\} \equiv \{\pi/4, 1, 10, 10^3\}$  and a controller sampling frequency  $f_c = 1$  kHz.

In order to study the system dynamics, during the contact we apply, separately, rectangular pulses, at the position and force references, that is, we perturb the reference with  $\{\delta y_{cd}, \delta F_{cd}\} = \{10^{-1}, 0\}$ .

#### 4.1. Time Response

**Figures 4 and 5** depict the time response for the position/force HC and the **Figs.6 and 7** for the cascade controller both under the action of the FO or PD/PI algorithms. These figures reveal that the FO algorithm is more stable than the PD/PI.

We analyze the response of a 2R robot with dynamic backlash at the joints [8, 10]. For the  $i$ th joint gear, with clearance  $h_i$ , the backlash reveals impact phenomena between the inertias, which are modeled according with the Newton law yielding:

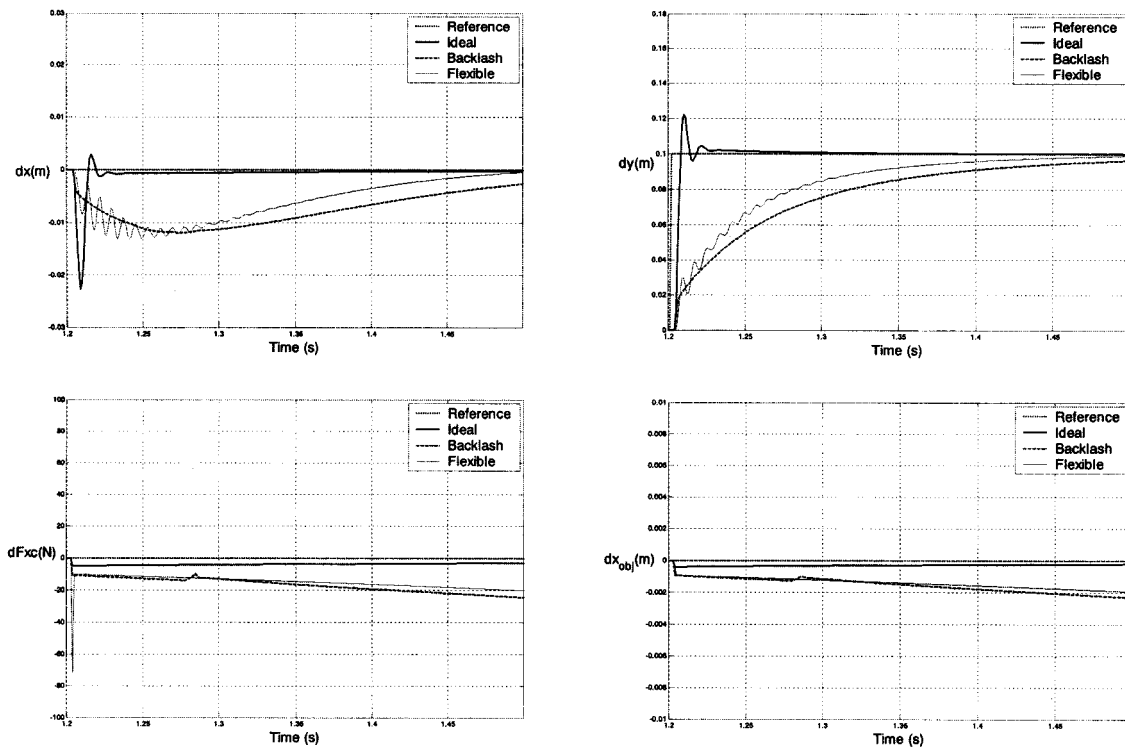
$$\dot{q}'_i = \frac{\dot{q}_i (J_{ii} - \varepsilon J_{im}) + \dot{q}_{im} J_{im} (1 + \varepsilon)}{J_{ii} + J_{im}} \quad \dots \quad (7a)$$

$$\dot{q}'_{im} = \frac{\dot{q}_i J_i (1 + \varepsilon) + \dot{q}_{im} (J_{im} - \varepsilon J_{ii})}{J_{ii} + J_{im}} \quad \dots \quad (7b)$$

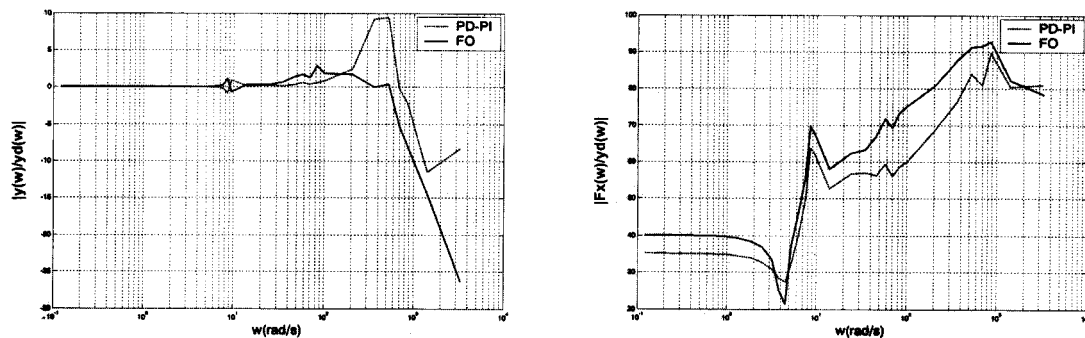
where  $0 \leq \varepsilon \leq 1$  is a constant that defines the type of impact ( $\varepsilon = 0$  inelastic impact,  $\varepsilon = 1$  elastic impact) and  $\dot{q}'_i$  and  $\dot{q}'_{im}$  are the inertias velocities of the joint and motor after the collision, respectively. The parameter  $J_{ii}(J_{im})$  stands for the link (motor) inertias of joint  $i$ . The numerical values adopted are  $h_i = 1.8 \times 10^{-4}$  rad and  $\varepsilon_i = 0.8 (i = 1, 2)$ .

We consider also 2R robot with compliant joints. For this case the dynamics corresponds to model (1) augmented by the equations:

$$\tau = \mathbf{J}_m \ddot{\mathbf{q}}_m + \mathbf{B}_m \dot{\mathbf{q}}_m + \mathbf{K}_m (\mathbf{q}_m - \mathbf{q}) \quad \dots \quad (8a)$$



**Fig. 7.** Time response for the 2R ideal robot and robots with joints having backlash and flexibility under the action of the *FO* algorithm for the *CC* and  $\delta y_d = 10^{-3}$  m.



**Fig. 8.** Frequency responses for the ideal robot under the action of the *FO* and the *PD/PI* controllers for the *HC* and  $\delta y_d = 10^{-3}$  m.

$$\mathbf{K}_m(\mathbf{q}_m - \mathbf{q}) = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \quad (8b)$$

where  $\mathbf{J}_m$ ,  $\mathbf{B}_m$  and  $\mathbf{K}_m$  are the  $n \times n$  diagonal matrices of the motor and transmission inertias, damping and stiffness, respectively. In the simulations we adopt  $K_{mi} = 2 \times 10^6 \text{ Nm} \cdot \text{rad}^{-1}$  and  $B_{mi} = 10^4 \text{ Nms} \cdot \text{rad}^{-1}$  ( $i = 1, 2$ ).

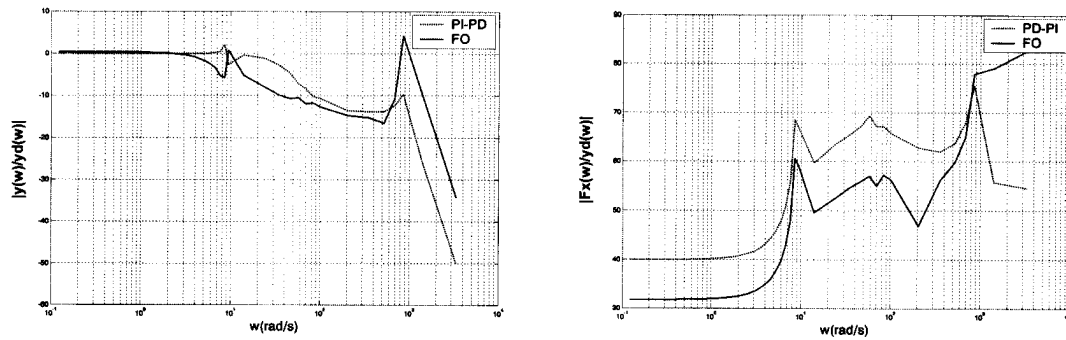
Figures 4 and 6 reveals that the *FO* algorithm is superior to the *PD/PI* in the cases with dynamical phenomena at the joints [8, 9].

To compare the *HC* and the *CC* architectures, we repeat the experiment for the *FO* and *PD/PI* algorithms, and we analyse the variation of the position  $\{x, y\}$  and force  $F$  of the robot and also the variation of the surface restriction  $x_{Cobj}$ .

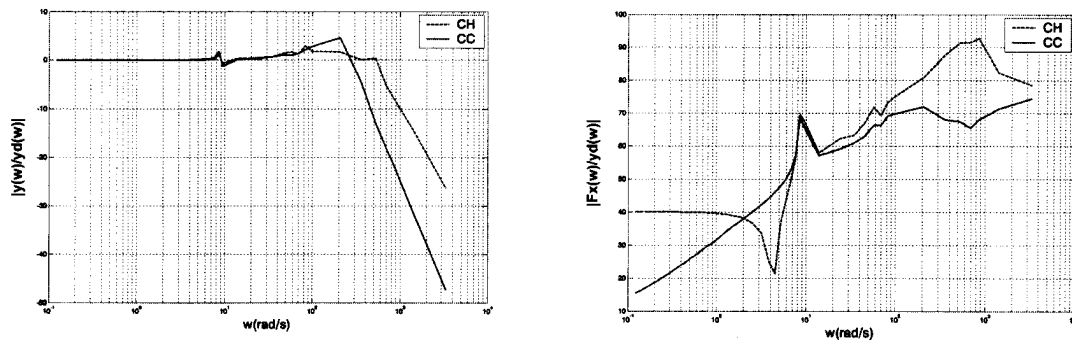
## 4.2. Frequency Response

Figures 8-10 show the transfer functions  $|Y_c(j\omega)/Y_{cd}(j\omega)|$ ,  $|F_c(j\omega)/F_{cd}(j\omega)|$ ,  $|Y_c(j\omega)/F_{cd}(j\omega)|$  and  $|F_c(j\omega)/Y_{cd}(j\omega)|$  (where  $Y_c(j\omega) = F\{\delta y_c\}$  and  $F_c(j\omega) = F\{\delta F_c\}$ ) for the *FO* and the *PD/PI* controllers, in the cases of an ideal robot and a robot with flexibility at the joints, respectively. The Charts reveal the existence of some coupling between the position and force loops due to the non-ideal performance of both algorithms.

Figures 8 and 10 show the frequency responses for robots with ideal joints and transmissions having flexibility, both under the action of the *FO* and the *PD/PI* controllers, for a pulse perturbation, at the robot reference  $\delta y_d$ . We can see that the system with the *PD/PI* con-



**Fig. 9.** Frequency responses for the robot with joints having flexibility, under the action of the *FO* algorithm, for the *HC* and  $\delta y_d = 10^{-3}$ m.



**Fig. 10.** Frequency responses for the ideal robot under the action of the *FO* algorithm, for the *HC* and *CC* architectures and  $\delta y_d = 10^{-3}$ m.

trollers has an higher overshoot and that the effect of the joint flexibility at the high frequencies.

The charts reveal that the *FO* algorithms have a superior performance, namely a good robustness and larger bandwidth.

## 5. Summary and Conclusions

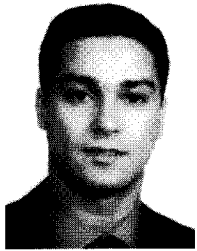
This paper presented the implementation of hybrid and cascade controllers for manipulators with several types of nonlinear phenomena at the joints. The system was tested both for fractional and integer order control algorithms.

The results show that the fractional order control algorithms have superior performances. Moreover, the cascade algorithm method is more stable than the hybrid position/force scheme.

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