

SINGLE-OBJECTIVE MAXIMIN SORTING SCHEME: APPLICATION TO RADIO FREQUENCY CIRCUIT DESIGN

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ABSTRACT

Obtaining a well distributed non-dominated Pareto front is one of the key issues in multi-objective optimization algorithms. This paper proposes an algorithm which promotes well distributed non-dominated fronts in the parameters space when a single-objective function is optimized. The proposed technique is described and tested in an automated synthesis circuit design problem. The project consists in designing CMOS radio-frequency and microwave binary-weighted differential switched capacitor arrays (RFDSCAs) from user top-level specification to component size. The genetic synthesis tool optimizes a fitness function based on the performance parameter of the RFDSCAs. To validate the proposed design methodology, a CMOS RFDSCA is synthesized, using a 0.25 μm BiCMOS technology, and verified by the SpectreRF simulator of the Cadence design environment. The results show that the synthesis and simulation outcomes are in very good agreement.

1. INTRODUCTION

Achieving a well-spread and well-diverse parameter front can be a time consuming computational problem. Some methods have been proposed in which a sharing model is used in order to obtain a set of optimal solutions dispersed along the optimal front. This technique was originally suggested by Goldberg and Richardson [1] where the fitness of similar solutions is degraded according to the distance of its closest neighbors. This technique will promote solutions in less crowded regions of the parameters space ‘forcing’ the population to be well distributed. Crowding techniques, introduced by De John [2], replace solutions by ‘similar’

ones. ‘Similarity’ is measured, like in the sharing model, by evaluating the distance between two solutions.

Maximin is a well known method used in classic multi-attribute problems [3] and in game theory [4, 5]. Recently, Balling [6] proposed a multi-objective optimization technique based on a fitness function derived from the maximin strategy [4], and Li [7] used the maximin fitness in a particle swarm multi-objective optimizer. Pires *et. al* [8] used a maximin technique to find a well distributed non-dominated Pareto front in multi-objective optimization algorithms.

Radio-frequency switched capacitor arrays (RFSCAs) in radio-frequency (RF) and microwave Si-integrated circuits and systems have been growing in importance since they were proposed in 1998 [9]. Nowadays, this kind of circuits is widely used, *e.g.* in multi-standard low-phase-noise ultra-wide-band voltage controlled oscillators [10], in low-noise fast-settling frequency synthesizers [11], in process dispersion compensation techniques [12] and in adaptive impedance matching circuits [13]. Furthermore, the RFSCAs have a great usage potential in reconfigurable or adaptive RF circuits for multi-mode, multi-band and multi-standard wireless transceivers. In the last years, the growing demand for full-integrated CMOS high performance wireless systems has spurred the researchers worldwide to develop new innovative techniques to optimize and automate the design of RF circuits [14, 15, 16, 17, 18, 19]. Although there is a large amount of published research work on this particular topic, no systematic design approach to synthesize optimum performance RFSCAs is known, in spite of the extensively use of RFSCAs in RF integrated tuned circuits. Bearing these ideas in mind, a method based on genetic algorithms to automate the design of optimum performance CMOS or BiCMOS RFDSCAs was developed. The

proposed method improves the performance, reliability and time-to-market (by reducing substantially the design time), since it eliminates the non-optimum trial-and-error design approach which is based on several rules of thumb.

This paper proposes a single-objective genetic algorithm, which promotes a solutions diversity in the parameters space, and applies the proposed technique to a radio frequency circuit design problem. Section 2 describes the proposed method and section 3 presents the RFDSCA circuit and its model. Besides that, the initial restrictions and objective function for this type of RF circuits are also defined in section 3. Section 4 shows the results carried out by the proposed algorithm and discusses them. Finally, section 5 outlines the main conclusions.

2. SINGLE-OBJECTIVE MAXIMIN SORTING SCHEME

This section presents the maximin sorting algorithm to render the following generation of GA. As it will be shown later, this technique has a good solution distribution in the parameters space.

The ϵ -dominance concept [20, 21] is used to obtain a solutions diversity over the objective space when solving some multi-objective problems. The proposed algorithm uses this concept and the maximin technique as the main ideas to achieve good diversity in the parameter space. Initially, the objective space is divided in several ranks, being each one characterized with a ϵ -distance (Figure 1). Inside each rank, the solutions have the same preference, even if their objective value is different. In a second phase, the best n distributed solutions are selected from a rank with m solutions ($m > n$). Therefore, to select a population of pop_{dim} solutions, the algorithm begins by selecting the solutions with lower rank (rank 1 of Figure 1) until the last allowed rank is considered and there are more solutions in this rank than the remaining slots in the population. In this case we select the best dispersed solutions of that rank.

The main concept behind the maximin sorting scheme is to select the solutions in order to decrease the large gap areas existing in the already selected population. For example, let us consider the solutions of one rank depicted in figure 2. In this case two parameters $\{x_1, x_2\}$ are considered. Initially the two extreme solutions of each parameter are selected, $\{a, b\}$ and $\{d, c\}$ for x_1 and x_2 , respectively. Through this selection the set $S \equiv \{a, b, c, d\}$ is initialized. Then, solution e is selected because it has the greater distance to the set S . After that, solution f is selected for inclusion into the set $S \equiv \{a, b, c, d, e\}$, for the same reasons. The process is repeated until S is completed.

The maximin sorting scheme is depicted in algorithm 1 and Table 1 presents the adopted notation. In each generation the new population is merged with their progenitor

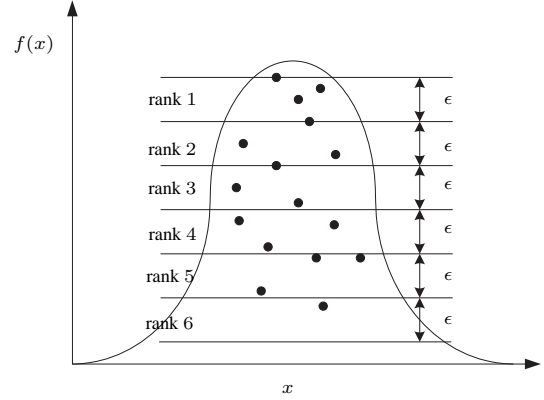


Fig. 1. Problem with one objective and one parameter x .

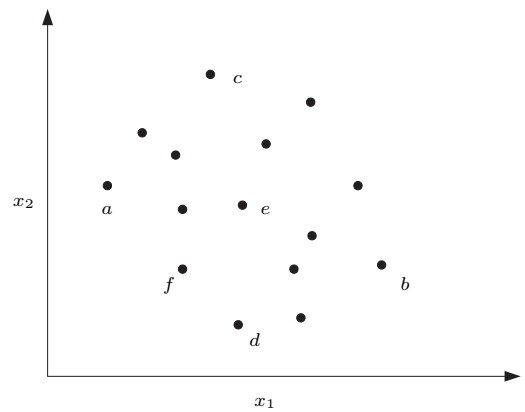


Fig. 2. Example of solutions in a bidimensional parameter space.

population, resulting a set R , from which the new set is obtained by applying the proposed algorithm (line 2). After that, the algorithm may select the extreme solutions from R for each parameter (lines 6-8) and introduces them into the set S . Then the individuals of lower rank are removed from the population T and inserted into the set S until the solutions number of the current rank surpass the allowed number of solutions of set S (lines 10-13). After that, the distance squared, c_{a_j} (1a), between each rank solution, a_j , and the solutions already selected, s_i , is evaluated. Then, the solution a_j , whose distance squared to the set S is the larger is selected (1b). Each time a solution enters into the set S , the cost c_{a_i} of the rank A is reevaluated (lines 20-22). This process ends when the set S is completed.

$$c_{a_j} = \min_{s_i \in S, a_j \in A} \|a_j - s_i\|^2 \quad (1a)$$

$$S = S \cup \{a_j : c_{a_j} = \max_{a_i \in A} c_{a_i}\} \quad (1b)$$

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1 begin
2    $R = P \cup D$ ;
3    $S = \emptyset$ ;
4    $A = \text{getRankMin}(R)$ ;
5   if  $\#A > \text{pop}_{dim}$  then
6     for  $i=1$  to  $n_{Par}$  do
7        $S = S \cup \text{getMin}(R, i) \cup \text{getMax}(R, i)$ 
8     end
9   end
10  while  $\#S + \#A \leq \text{pop}_{dim}$  do
11     $S = S \cup A$ ;
12     $A = \text{getRankMin}(R)$ 
13  end
14  for  $j = 1$  to  $\#A$  do
15     $c_{a_j} = \min_{s_i \in S} \{\|a_j - s_i\|^2\}$ 
16  end
17  while  $\#S < \text{pop}_{dim}$  do
18     $k = \text{getMaxCi}(A)$ ;
19     $S = S \cup k$ ;
20    for  $l = 1$  to  $\#A$  do
21       $c_{a_l} = \min\{\|a_l - k\|^2, a_l\}$ 
22    end
23  end
24 end

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Algorithm 1: Single-objective maximin algorithm.

3. AUTOMATED CIRCUIT DESIGN ALGORITHM

To access the algorithm performance, an application example in the area of RF integrated circuits is used. In this sense, the optimization procedure is employed to automate the design of a RFDSCA circuit with the goal of finding the component values that maximize the RFDSCA performance. The circuit is developed in a $0.25 \mu\text{m}$ BiCMOS technology and it is intended to be a cell of a ku-band voltage controlled oscillator (VCO).

The topology of the RFDSCA (Figure 3) consists of N cells, each one constituted by two cell capacitors and a cell switch. The first cell, named reference cell, is comprised by two reference capacitors, both with value C , and a reference switch. This switch is formed by placing in parallel M basic switches (BS). For the other cells, the capacitors values are equal to $2^{i-1}C$ and the number of reference switches for each cell switch is given by 2^{i-1} , where i is the cell number [22, 23].

To achieve high performance RFDSCAs, during the circuit design phase, it is necessary to obtain the components that maximizes the RFDSCA quality factor (Q_{RFDSCA}). This performance parameter is given by the following equation:

Table 1. Description of variables and functions used in maximin algorithm.

	Description
P	Parent population
D	Offspring population
A, R	Auxiliary populations
S	Final population
n_{Par}	Number of parameters
pop_{dim}	Population size
k	Solution with maximal squared distance
c_i	Minimum euclidian norm between solution i and set S
$\text{getMin}(X, i)$	Remove from set X the solution whose parameter i is minimal
$\text{getMax}(X, i)$	Remove from set X the solution whose parameter i is maximal
$\text{getMaxCi}(X)$	Remove the solution from set X whose c_i is maximum
$\text{getRankMin}(X)$	Remove all solutions with lower rank from set X

$$Q_{\text{RFDSCA}}(D, f) = \frac{\left(1 + \frac{2M C_{\text{BS-OFF}}}{C}\right) \left(1 + \frac{D}{D_{\text{max}}} \frac{C}{2M C_{\text{BS-OFF}}}\right)}{1 + \frac{D}{D_{\text{max}}} \left[\left(\frac{2M C_{\text{BS-OFF}}}{C} + 1\right)^2 \frac{R_{\text{BS-ON}}}{R_{\text{BS-OFF}}} - 1 \right]} \quad (2)$$

where D is the control word (decimal representation of the control binary word, $D = b_N 2^{N-1} + \dots + b_2 2^1 + b_1 2^0$), f is the operating frequency and $D_{\text{max}} = 2^{N-1}$ corresponds to the maximum value of D . The $R_{\text{BS-ON}}$, $R_{\text{BS-OFF}}$ is the ON and OFF resistance of the basic switch, respectively. $C_{\text{BS-OFF}}$ is its OFF capacitance.

Observing that Q_{RFDSCA} decreases monotonically with f , the objective function for this kind of RF circuits can be

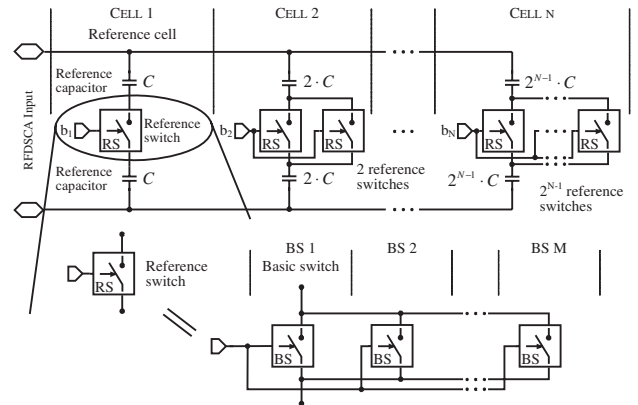


Fig. 3. Binary-weighted RFDSCA architecture.

made independent of it. Therefore, the chosen optimization objective function is given by:

$$f_v = 2\pi f Q_{\text{RFDSCA}} \quad (3)$$

To fulfill the required tuning range requirements for the VCO (13.5 GHz to 15.5 GHz), the discrete capacitance tuning circuit must present, at least, a minimum capacitance of 72 fF, a maximum capacitance of 108 fF and a maximum tuning step of 8 fF, as indicated by the following constraints:

$$\begin{cases} C_{\text{RFDSCA-MAX}} \geq C_{\text{max}} = 108\text{fF} \\ C_{\text{RFDSCA-MIN}} \leq C_{\text{min}} = 72\text{fF} \\ \Delta C_{\text{RFDSCA}} \leq \Delta C = 8\text{fF} \end{cases} \quad (4)$$

The variables $C_{\text{RFDSCA-MAX}}$, $C_{\text{RFDSCA-MIN}}$ and ΔC_{RFDSCA} represent the maximum RFDSCA capacitance, the minimum RFDSCA capacitance and the RFDSCA tuning capacitance step, respectively. These three capacitance variables can be calculated from the following equation:

$$C_{\text{RFDSCA}}(D) = \frac{D \frac{C}{2} + D_{\text{max}} \frac{MC_{\text{BS-OFF}}}{C}}{1 + \frac{2MC_{\text{BS-OFF}}}{C}} \quad (5)$$

Considering the SPICE models of the NMOS transistors and the technological process parameters, the elements of the BS model present the following values: $R_{\text{BS-ON}} = 73.6\Omega @ V_{\text{GS}} = 3V$, $R_{\text{BS-OFF}} = 18.9\Omega @ V_{\text{GS}} = 0V$ and $C_{\text{BS-OFF}} = 6.1 \text{ fF} @ V_{\text{GS}} = 0V$.

The developed optimization algorithm is based on a GA with the above-mentioned maximin scheme, which uses 10^3 potential solutions, each one represented by the optimization parameters N , M and C . These floating point values are randomly initialized in an appropriate range ($N = 1 \dots 64$, $M = 1 \dots 64$ and $C = 1\text{fF} \dots 1\text{pF}$). The search is then carried out with this population over 10^7 generations. The fitness value, f_v , is given by (3) if the solution verifies the restrictions, otherwise takes a negative value, proportional to the distance to the feasible decision region, if at least one restriction is not satisfied. The successive generations of new solutions are reproduced based on a linear ranking scheme and simulated binary crossover [24]. Finally, when mutation occurs the operator replaces the value of one design parameter according to a uniform distribution function. The uniform function varies in the range $[-U, U]$ where $U = \{0.2, 0.2, 0.4 \times 10^{-15}\}$ for N , M and C , respectively. The crossover and mutation probabilities are $p_c = 0.6$ and $p_m = 0.05$, respectively. The height of each rank is $\epsilon = 10^{11}$.

4. AUTOMATED DESIGN RESULTS

For this case study, the RFDSCA circuits with optimum performance obtained by the GA can be directly defined by the

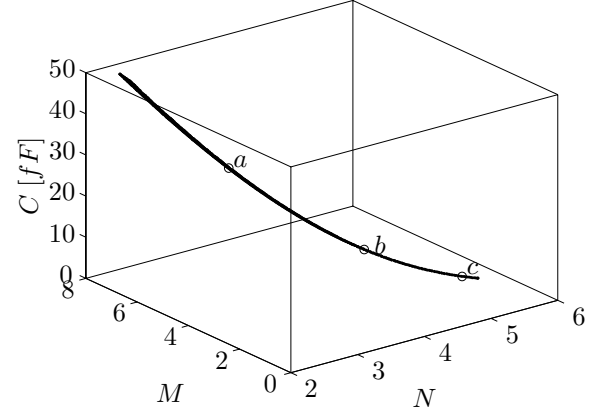


Fig. 4. Optimal parameters front in the $N \times M \times C$ space, $a = \{3.0, 5.0, 30.9\text{fF}\}$, $b = \{4.0, 2.3, 14.4\text{fF}\}$, $c = \{5.0, 1.1, 7.0\text{fF}\}$.

dot-points of figures 4 and 5. These were obtained using the initial specifications stated in (4) in the proposed automated design procedure. Figure 4 shows the optimum solution front in a 3D space and Figure 5 the corresponding projections in $N \times C$ and $M \times C$ planes. These two figures clearly show that the algorithm finds a front with good diversity. An important aspect about the algorithm performance is its convergence to the optimum solutions. This property can be inferred by the fitness value of each solution. Figure 6 presents the fitness f_v versus N , M and C for the current case study. These curves show that the algorithm convergence ability is very good since all the solutions are in the same rank. This is easily verified since the error between the best and worst fitness solutions are lower than $\epsilon = 10^{11}$. These charts reveal that the circuit can have several possible implementations with the same performance (the problem has not a single isolated optimum point, but, in fact an optimal front).

To find a RFDSCA circuit, it is only necessary to choose M , N and C from Figure 5. For instance, three possible circuits are flagged in Figure 4 as $\{a, b, c\}$. As it can be seen in Table 2, the solutions respect the design restrictions defined in (4). To infer the validity of the synthesis results, a comparison between the RFDSCA performance predicted by the automated design method and by the circuit simulation on SpectreRF is provided in the same table. The optimization objective shown in Table 2 is evaluated by the Q_{RFDSCA} , at an operation frequency of 15 GHz, when all the RFDSCA constitutive cells are in the ON state (when $D = D_{\text{max}}$). The RFDSCA performance obtained in SpectreRF, with the optimization variables defined in solution a , can be considered equal to the one obtained with the GA (Table 2). This happens because the values of the optimization variables N and M are almost integer numbers. A different situation occurs with the solutions b and c . In these two

Table 2. RFDSCA Performance results obtained with the maximin algorithm and with SpectreRF.

Solutions	Type of Results	N	M	C [fF]	Design Constrains [fF] @15GHz			Optimization Objective $Q_{\text{RFDSCA-ON}}(D = D_{\text{max}})$ (maximization)
					$C_{\text{RFDSCA-MAX}}$ ($\geq 108\text{fF}$)	$C_{\text{RFDSCA-MIN}}$ ($\leq 72\text{fF}$)	ΔC_{RFDSCA} ($\leq 8\text{fF}$)	
a	GA	2.998	5.026	30.93	108	71.95	5.185	46.83
	SpectreRF	3	5	30.90	108	71.70	5.185	46.7@15GHz
b	GA	4.0011	2.3476	14.40	108	71.90	2.408	46.89
	SpectreRF	4	2	14.40	108	67.92	2.670	40.0@15GHz
c	GA	4.998	1.1251	6.99	108	71.65	1.179	46.40
	SpectreRF	5	1	6.97	108	68.74	1.265	41.4@15GHz

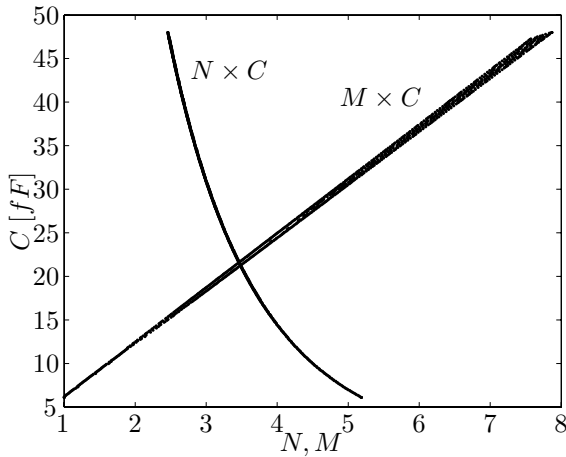


Fig. 5. Projections in the planes $N \times C$ and $M \times C$ of optimal front.

cases, although the N values provided by the GA can be considered integer numbers the same does not happens with M .

To simulate the circuit with the solutions b and c in SpectreRF, it is necessary to round M to the nearest integer number (solution b : $\text{round}(M = 2.3476) = 2$ and solution c : $\text{round}(M = 1.1251) = 1$), since it represents the number of BS that constitutes the reference switch. The performance comparison for the solutions b and c shows that the capacitive range and tuning step of the RFDSCA in the SpectreRF is similar to the one obtained with the synthesis procedure. Nevertheless, the number rounding process has some impact on the optimization objective, as shown in Table 2.

5. CONCLUSIONS

A synthesis procedure to automate the design of RFDSCAs is presented in this paper. The synthesis is carried on by a genetic algorithm that promotes the distribution of the solution along the parameters space in order to give several optimal solutions. This method is based on closed-form sym-

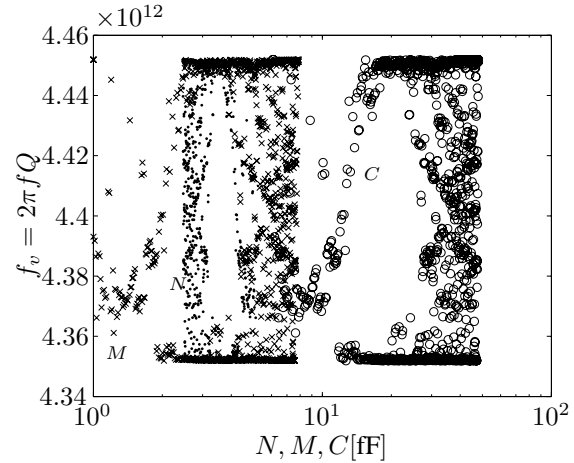


Fig. 6. Fitness f_v versus parameters $\{M - \text{x-points}, N - \text{dot-points}, C - \text{circle-points}\}$.

bolic mathematical expressions of the input impedance and quality factor of the RFDSCA. This means that a RFDSCA circuit can be implemented with different component sizes. To verify the proposed synthesis method, three RFDSCAs were designed using three different solutions provided by the GA for the same design constraints. The performance results of the three RFDSCAs, obtained with the GA, were then compared with the ones achieved by simulating them in the SpectreRF. The analysis shows that the GA is able to reach optimal solutions regarding the optimization objective. Moreover, the GA obtains a set of solutions along the optimal front in one run of the algorithm.

6. REFERENCES

- [1] D. E. Goldberg and J. Richardson, "Genetic algorithms with sharing for multi-modal function optimisation," in *Proc of the 2nd Int. Conf. on Genetic Algorithms and Their Applications*, 1987, pp. 41–49.
- [2] Kenneth Alan De Jong, *An Analysis of the Behavior of a Class of Genetic Adaptive Systems*, Ph.D. thesis, University of Michigan, 1975.

- [3] Ching-Lai Hwang K. Paul Yoon, *Multiple Attribute Decision Making : An Introduction (Quantitative Applications in the Social Sciences)*, SAGE Publications, 1995.
- [4] R. D. Luce and H. Raiffa, *Introduction and Critical Survey*, John Wiley & Sons, Inc, New York, 1957.
- [5] J. Rawls, *A Theory of Justice*, Oxford University Press, London, 1971.
- [6] Richard Balling, "The maximin fitness function; multi-objective city and regional planning," in *Evolutionary Multi-Criterion Optimization, Second International Conference, EMO 2003, Faro, Portugal, April 8-11, 2003, Proceedings*, Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, Eds. 2003, vol. 2632 of *Lecture Notes in Computer Science*, pp. 1–15, Springer.
- [7] X. Li, "Better spread and convergence: Particle swarm multiobjective optimization using the maximin fitness function," in *Proceeding of Genetic and Evolutionary Computation Conference 2004 (GECCO'04)*, K. et al. Deb, Ed., Seattle, USA, 26–30 June 2004, vol. LNCS 3102 of *Lecture Notes in Computer Science*, Springer-Verlag, pp. 117–128.
- [8] E. J. Solteiro Pires, P. B. de Moura Oliveira, and J. A. Tenreiro Machado, "Multi-objective maximin sorting scheme," in *Conference on Evolutionary Multi-criterion Optimization – EMO 2005*, Guanajuato, México, Mar 2005, pp. 165–175, Springer-Verlag, *Lecture Notes in Computer Science* Vol. 3410.
- [9] A. Kral, F. Behbahani, and A. Abidi, "RF-CMOS oscillators with switched tuning," in *IEEE CICC*, MAY 1998.
- [10] A. Fard, "Phase noise and amplitude issues of a wide-band VCO utilizing a switched tuning resonator," in *IEEE ISCAS*, May 2005.
- [11] S. Wu, "A low-noise fast-settling PLL frequency synthesizer for CDMA receivers," in *SoC*, November 2004.
- [12] K. Ang, M. Chia, and D. Li, "A process compensation technique for integrated VCO," in *IEEE RFIC*, June 2004.
- [13] P. Sjöblom and H. Sjöland, "An adaptive impedance tuning CMOS circuit for ISM 2.4-GHz band," *IEEE Trans. on Circ. and Syst.*, vol. 52, no. 6, pp. 1115–1124, June 2005.
- [14] A. Somani, P. P. Chakrabarti, and A. Patra, "An evolutionary algorithm-based approach to automated design of analog and RF circuits using adaptive normalized cost functions," *IEEE Trans. on Evol. Comp.*, In Press.
- [15] S. Kaitharam, C. Rajagopal, and A. Nunez, "SRFCC: synthesis of RF CMOS circuits," in *IEEE MWSCAS*, August 2002.
- [16] C. Rajagopal, K. Sridhar, and A. Nunez, "RF CMOS circuit optimizing procedure and synthesis tool," in *ACM VLSI*, April 2003.
- [17] G. Konstantopoulos, K. Papathanasiou, and A. Samelis, "Optimization of RF circuits by expert system monitored genetic computation," in *IEEE ISCAS 2006*, May 2006.
- [18] M. Chu, D. J. Allstot, J. M. Huard, and K. Y Wong, "NSGA-based parasitic-aware optimization of a 5 GHz low-noise VCO," in *IEEE ASP DAC*, January 2004.
- [19] P. Sen, R. J. Pratap, R. Mukhopadhyay, S. Sarkar, C. H. Lee, S. Pinel, G. S. May, and J. Laskar, "Neuro-genetic design centering of millimeter wave oscillators," in *IEEE SiRF*, January 2006.
- [20] M. Laumanns, L. Thiele, K. Deb, and E. Zitzler, "Archiving with guaranteed convergence and diversity in multi-objective optimization," in *In Proceedings of the Genetic and Evolutionary Computation Conference Morgan Kaufmann Publishers*, 2002, pp. 439–447.
- [21] Kalyanmoy Deb, Manikanth Mohan, and Shikhar Mishra, "A fast multi-objective evolutionary algorithm for finding well-spread pareto-optimal solutions," KanGal Report 2003002, Indian Institute of Technology Kanpur, February 2003.
- [22] L. Mendes, J. C. Vaz, and M. J. Rosário, "A closed-form input admittance solution for RF and microwave switched capacitor arrays," in *IEEE MWSCAS*, San Juan, Puerto Rico, 6-9 August 2006.
- [23] L. Mendes, J. C. Vaz, and M. J. Rosário, "Performance of Si-integrated wide-band single-ended switched capacitor arrays," in *13th IEEE ICECS*, Nice, France, December 2006, pp. 10–13.
- [24] Kalyanmoy Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, LTD, 2001.