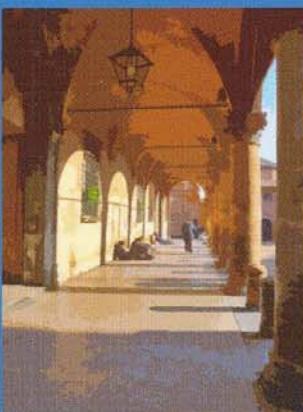


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# FRACTIONAL $PD^\alpha$ CONTROL OF AN HEXAPOD ROBOT

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**Abstract** - This paper studies the performance of a Fractional Order  $PD^\alpha$  controller in a hexapod robot with three dof legs and leg joint actuators having saturation. For that objective the robot prescribed motion is characterized in terms of several locomotion variables. Moreover, two indices measure the walking performance based on the mean absolute density of energy per travelled distance and on the hip trajectory errors. A set of simulation experiments reveals the influence of the different controller tuning upon the proposed indices. *Copyright © 2006 IFAC*

**Keywords** – Robotics, Walking, Control algorithms, Performance analysis

## 1. INTRODUCTION

Walking machines allow locomotion in terrain inaccessible to other type of vehicles, since they do not need a continuous support surface. On the other hand, the requirements for leg coordination and control impose difficulties beyond those encountered in wheeled robots. There exists a class of walking machines for which locomotion is a natural dynamic mode. Once started on a shallow slope, a machine of this class will settle into a steady gait, without active control or energy input. However, the capabilities of these machines are quite limited.

Previous studies focused mainly in the control at the leg level and leg coordination using neural networks, fuzzy logic, central pattern generators and subsumption architecture. There is also a growing interest in using insect locomotion schemes to control walking robots (Silva and Machado, 2006b). In spite of the diversity of approaches, for multi-legged robots the control at the joint level is usually implemented through a simple PID like scheme with position/velocity feedback. Other approaches include sliding mode control (Martins-Filho, *et al.*, 2003), computed torque control (Lee, *et al.*, 1998) and hybrid force/position control (Song, *et al.*, 1999).

The application of the theory of fractional calculus in robotics is still in a research stage, but the recent progress in this area reveals promising aspects for future developments (Silva, *et al.*, 2003a).

With these facts in mind, a simulation model for

multi-leg locomotion systems was developed, for several periodic gaits. Based on this tool, the present study compares different Fractional Order (FO) robot controller tuning. The analysis is based on the formulation of two indices measuring the mean absolute density of energy per travelled distance and the hip trajectory errors during walking. It is analysed the system performance for two cases: two leg joints are motor actuated and the ankle joint is mechanical actuated and the three leg joints are fully motor actuated. The simulations reveal the superior performance of the FO controller, with all leg joints motor actuated.

Bearing these facts in mind, the paper is organized as follows. Section two introduces the robot kinematic model and the motion planning scheme. Sections three and four present the robot dynamic model and control architecture and the optimizing indices, respectively. Section five develops a set of simulation experiments that compare the performance of the different controller tuning. Finally, section six outlines the main conclusions and directions towards future developments.

## 2. ROBOT KINEMATICS AND TRAJECTORY PLANNING

We consider a walking system (Fig. 1) with  $n = 6$  legs, equally distributed along both sides of the robot body, having each three rotational joints (*i.e.*,  $j = \{1, 2, 3\} \equiv \{\text{hip, knee, ankle}\}$ ) (Silva, *et al.*, 2006a).



where, for leg  $i$  and joint  $j$ ,  $\tau_{ijC}$  is the controller demanded torque,  $\tau_{ijMax}$  is the maximum torque that the actuator can supply and  $\tau_{ijm}$  is the motor effective torque.

### 3.2 Joint $j = 3$ Implementation

During this study leg joint  $j = 3$  can be either mechanical actuated or motor actuated. For the mechanical actuated case, we suppose that there is a rotational pre-tensioned spring-dashpot system connecting leg links  $L_{i2}$  and  $L_{i3}$ . This mechanical impedance maintains the angle between the two links and imposes a joint torque given by (for leg  $i$ ):

$$\begin{aligned}\tau_{i3m} &= K_3 \Delta_{i3} + B_3 \dot{\Delta}_{i3} \\ \Delta_{i3} &= \theta_{i3d}(t) - \theta_{i3}(t), \dot{\Delta}_{i3} = \dot{\theta}_{i3d}(t) - \dot{\theta}_{i3}(t)\end{aligned}\quad (7)$$

where,  $\tau_{i3m}$  is the joint effective torque,  $K_3$  and  $B_3$  are the coefficients of stiffness and viscous friction and  $\theta_{i3d}$  and  $\theta_{i3}$  are the planned and real joint trajectories.

### 3.3 Robot Body Model

Figure 2 presents the dynamic model for the hexapod body and foot-ground interaction. It is considered a robot body compliance because walking animals have a spine that allows supporting the locomotion with improved stability. In the present study, the robot body is divided in  $n$  identical segments (each with mass  $M_b n^{-1}$ ) and a linear spring-damper system is adopted to implement the intra-body compliance:

$$f_{i\eta H} = \sum_{i'=1}^u \left[ -K_{\eta H} (\eta_{iH} - \eta_{i'H}) - B_{\eta H} (\dot{\eta}_{iH} - \dot{\eta}_{i'H}) \right] \quad (8)$$

where  $(x_{iH}, y_{iH})$  are the hip coordinates and  $u$  is the total number of segments adjacent to leg  $i$ .

In this study, the parameters  $K_{\eta H}$  and  $B_{\eta H}$  ( $\eta = \{x, y\}$ ) in the {horizontal, vertical} directions, respectively, are defined so that the body behaviour is similar to the one expected to occur on an animal (Table 1).

### 3.4 Foot-Ground Interaction Model

The contact of the  $i^{\text{th}}$  robot feet with the ground is modelled through a non-linear system (Silva, *et al.*, 2003b) with linear stiffness  $K_{\eta F}$  and non-linear damping  $B_{\eta F}$  ( $\eta = \{x, y\}$ ) in the {horizontal, vertical} directions, respectively (see Fig. 2), yielding:

$$\begin{aligned}f_{i\eta F} &= -K_{\eta F} (\eta_{iF} - \eta_{iF0}) - B_{\eta F} \left[ -(y_{iF} - y_{iF0}) \right]^{v_\eta} (\dot{\eta}_{iF} - \dot{\eta}_{iF0}) \\ v_x &= 1.0, v_y = 0.9\end{aligned}\quad (9)$$

where  $x_{iF0}$  and  $y_{iF0}$  are the coordinates of foot  $i$  touchdown and  $v_\eta$  is a parameter dependent on the ground characteristics. The values for the parameters  $K_{\eta F}$  and  $B_{\eta F}$  (Table 1) are based on the studies of soil mechanics (Silva, *et al.*, 2003b).

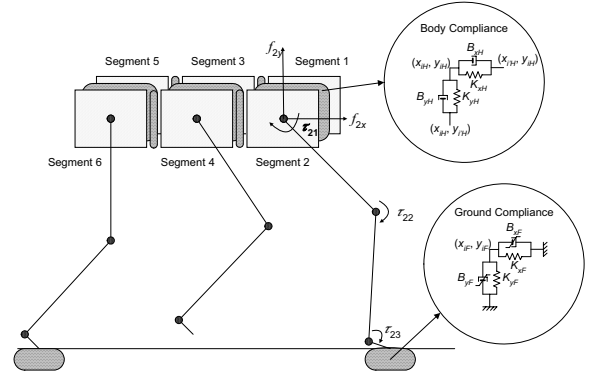


Fig. 2. Model of the robot body and foot-ground interaction.

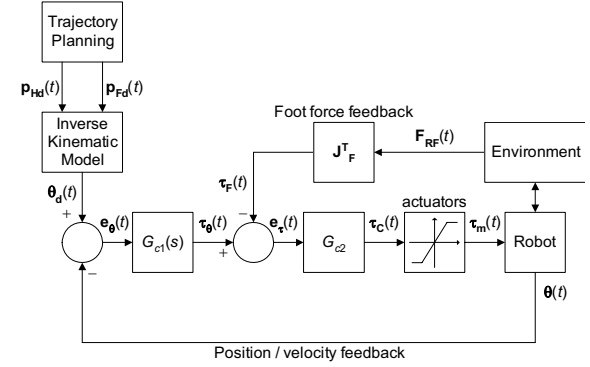


Fig. 3. Hexapod robot control architecture.

Table 1 System parameters

Robot model parameters		Locomotion parameters	
$S_P$	1 m	$\beta$	50%
$L_{ij}, j=1,2$	0.5 m	$L_S$	1 m
$L_{i3}$	0.1 m	$H_B$	0.9 m
$O_i$	0 m	$F_C$	0.1 m
$M_b$	88.0 kg	$V_F$	1 ms <sup>-1</sup>
$M_{ij}, j=1,2$	1 kg	Ground parameters	
$M_{i3}$	0.1 kg	$K_{xF}$	$1.3 \times 10^6$ Nm <sup>-1</sup>
$K_{xH}$	$10^5$ Nm <sup>-1</sup>	$K_{yF}$	$1.7 \times 10^6$ Nm <sup>-1</sup>
$K_{yH}$	$10^4$ Nm <sup>-1</sup>	$B_{xF}$	$2.3 \times 10^6$ Nsm <sup>-1</sup>
$B_{xH}$	$10^3$ Nsm <sup>-1</sup>	$B_{yF}$	$2.7 \times 10^6$ Nsm <sup>-1</sup>
$B_{yH}$	$10^2$ Nsm <sup>-1</sup>		

### 3.5 Control Architecture

The general control architecture of the hexapod robot is presented in Fig. 3. On a previous work were demonstrated the advantages of a cascade controller, with PD position control and foot force feedback, over a classical PD with, merely, position feedback, particularly in real situations where we have non-ideal actuators with saturation and being also more robust for variable ground characteristics (Silva, *et al.*, 2003b). Based on these results, in this study we evaluate the effect of different FO PD <sup>$\alpha$</sup>  controller implementations for  $G_{c1}(s)$ , while for  $G_{c2}$  it is considered a simple P controller. For the FO PD <sup>$\alpha$</sup>  algorithm we have:

$$G_{c1j}(s) = Kp_j + K\alpha_j s^{\alpha_j}, \quad \alpha_j \in \mathbb{R}, \quad j = 1, 2, 3 \quad (10)$$

where  $Kp_j$  and  $K\alpha_j$  are the proportional and derivative gains, respectively, and  $\alpha_j$  is the fractional order, for joint  $j$ . Therefore, the classical PD<sup>1</sup> algorithm occurs when the fractional order  $\alpha_j = 1.0$ .

In what concerns Eq. (10) it should be noted that the mathematical definition of a derivative of fractional order has been the subject of several different approaches (Machado, 1997). For example, Eq. (11a) and Eq. (11b), represent the Laplace (for zero initial conditions) and the Grünwald-Letnikov definitions of the fractional derivative of order  $\alpha$  of the signal  $x(t)$ :

$$D^\alpha[x(t)] = L^{-1}\{s^\alpha X(s)\} \quad (11a)$$

$$D^\alpha[x(t)] = \lim_{h \rightarrow 0} \left[ \frac{1}{h^\alpha} \sum_{k=1}^{\infty} \frac{(-1)^k \Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)} x(t-kh) \right] \quad (11b)$$

where  $\Gamma$  is the gamma function and  $h$  is the time increment.

In this paper, for implementing the FO algorithm (Eq. (10)) it is adopted a discrete-time 4<sup>th</sup>-order Padé approximation ( $a_{ij}, b_{ij} \in \mathbb{R}, j = 1, 2, 3$ ) yielding an equation in the  $z$ -domain of the type:

$$G_{C1j}(z) \approx Kp_j + K\alpha_j \sum_{i=0}^{i=4} a_{ij} z^{-i} / \sum_{i=0}^{i=4} b_{ij} z^{-i}. \quad (12)$$

#### 4. MEASURES FOR PERFORMANCE EVALUATION

In mathematical terms we establish two global measures of the overall performance of the mechanism in an average sense. In this perspective, we define one index  $\{E_{av}\}$  inspired on the system dynamics and another one  $\{\varepsilon_{xyH}\}$  based on the trajectory tracking errors.

Regarding the mean absolute density of energy per travelled distance  $E_{av}$ , it is computed assuming that energy regeneration is not available by actuators doing negative work (by taking the absolute value of the power). At a given joint  $j$  (each leg has  $m=3$  joints) and leg  $i$  (since we are adopting a hexapod it yields  $n=6$  legs), the mechanical power is the product of the motor torque and angular velocity. The global index  $E_{av}$  is obtained by averaging the mechanical absolute energy delivered over the travelled distance  $d$ :

$$E_{av} = \frac{1}{d} \sum_{i=1}^n \sum_{j=1}^m \int_0^T |\tau_{ij}(t) \dot{\theta}_{ij}(t)| dt \quad [\text{Jm}^{-1}] \quad (13)$$

In what concerns the hip trajectory following errors we can define the index:

$$\varepsilon_{xyH} = \sum_{i=1}^n \sqrt{\frac{1}{N_s} \sum_{k=1}^{N_s} (\Delta_{ixH}^2 + \Delta_{iyH}^2)} \quad [\text{m}] \quad (14)$$

$$\Delta_{ixH} = x_{iHd}(k) - x_{iH}(k), \Delta_{iyH} = y_{iHd}(k) - y_{iH}(k)$$

where  $N_s$  is the total number of samples for averaging purposes and  $\{d, r\}$  indicate the  $i^{\text{th}}$  samples of the desired and real position, respectively.

In all cases the performance optimization requires the minimization of each index.

Table 2 Controller parameters for joint 3 mechanical actuated

$\alpha_j$	$Kp_1$	$K\alpha_1$	$Kp_2$	$K\alpha_2$	$K_3$	$B_3$
0.4	10000.0	3200.0	800.0	300.0	2.0	0.5
0.5	15000.0	6000.0	1000.0	600.0	0.5	2.0
0.6	2500.0	800.0	300.0	100.0	1.0	2.0
0.7	2000.0	500.0	400.0	100.0	0.5	0.5
0.8	2000.0	400.0	300.0	100.0	4.0	3.5
1.0	8000.0	60.0	500.0	40.0	5.0	2.5

Table 3 Controller parameters for joint 3 motor actuated

$\alpha_j$	$Kp_1$	$K\alpha_1$	$Kp_2$	$K\alpha_2$	$Kp_3$	$K\alpha_3$
0.4	8000.0	2900.0	900.0	400.0	100.0	80.0
0.5	15000.0	7200.0	1000.0	800.0	150.0	240.0
0.6	600.0	150.0	250.0	40.0	100.0	15.0
0.7	600.0	150.0	150.0	15.0	80.0	15.0
0.8	500.0	80.0	200.0	30.0	80.0	10.0
1.0	8000.0	60.0	500.0	40.0	100.0	2.5

#### 5. SIMULATION RESULTS

In this section we develop a set of simulations to analyse the performances of the different FO PD $^\alpha$  controller tuning during a periodic wave gait at a constant forward velocity  $V_F$ . For simulation purposes we consider the locomotion parameters, the robot body parameters and the ground parameters (supposing that the robot is walking on a ground of compact clay) presented in Table 1.

To tune the different controller implementations we adopt a systematic method, testing and evaluating several possible combinations of parameters, for all controller implementations. Therefore, we adopt the  $G_{c1}(s)$  parameters that establish a compromise in what concerns the simultaneous minimisation of  $E_{av}$  and  $\varepsilon_{xyH}$ , and a proportional controller  $G_{c2}$  with gain  $Kp_j = 0.9$  ( $j = 1, 2, 3$ ). Moreover, it is assumed high performance joint actuators, with a maximum actuator torque in Eq. (6) of  $\tau_{ijMax} = 400$  Nm, and  $\theta_{i3hd} = -15^\circ$ . We start by considering that leg joints 1 and 2 are motor actuated and joint 3 is mechanical actuated. For this case we tune the FO PD $^\alpha$  joint controllers for different values of the fractional order  $\alpha_j$  in the interval  $-0.9 < \alpha_j < +0.9$  and  $\alpha_j \neq 0.0$ . For comparison purposes we also consider the classical PD algorithm. Afterwards, we consider that joint 3 is also motor actuated, and we repeat the controller tuning procedure versus  $\alpha_j$ . The controller parameters, for both cases, are presented in Tables 2 and 3.

Figure 4 presents the best controller tuning for different values of  $\alpha_j$  when joint 3 is mechanical actuated. We observe that the value of  $\alpha_j = 0.6$  presents the best compromise situation in what concerns the simultaneous minimisation of  $\varepsilon_{xyH}$  and  $E_{av}$ . For values of  $\alpha_j = \{0.4, 0.5, 0.7, 1.0\}$  the values of  $\varepsilon_{xyH}$  are similar and slightly higher than the corresponding value for  $\alpha_j = 0.6$ . For  $\alpha_j = 0.8$  the index  $\varepsilon_{xyH}$  yields much higher values. Concerning the values of  $E_{av}$ , the minimum is obtained for  $\alpha_j = 0.6$ , while increasing for  $\alpha_j = 0.5$  and  $\alpha_j = 0.4$ . For  $\alpha_j = \{0.7, 0.8, 1.0\}$  the values of  $E_{av}$  are much higher and are not presented in Figure 6.

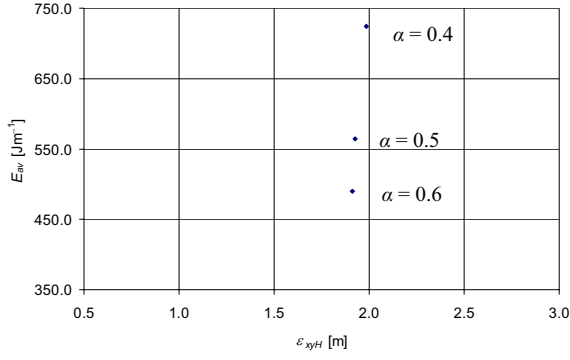


Fig. 4. Plots of  $E_{av}$  vs.  $\varepsilon_{xyH}$  for the different  $G_{c1}(s)$  FO controller tuning, when establishing a compromise between the minimisation of  $E_{av}$  and  $\varepsilon_{xyH}$ , with  $G_{c2} = 0.9$ , joints 1 and 2 motor actuated and joint 3 mechanical actuated.

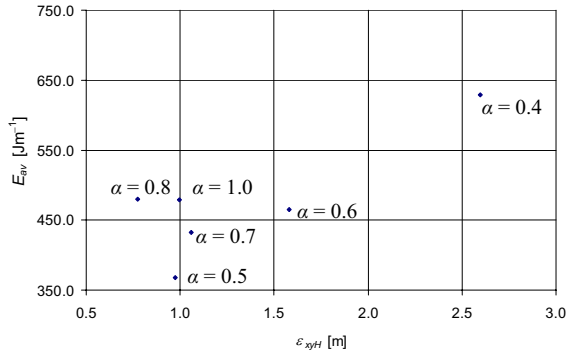


Fig. 5. Plots of  $E_{av}$  vs.  $\varepsilon_{xyH}$  for the different  $G_{c1}(s)$  FO controller tuning, when establishing a compromise between the minimisation of  $E_{av}$  and  $\varepsilon_{xyH}$ , with  $G_{c2} = 0.9$  and all joints motor actuated.

Figure 5 depicts a similar chart for the case when all joints are motor actuated. We observe that the value of  $\alpha_j = 0.5$  presents the best compromise situation in what concerns the simultaneous minimisation of  $\varepsilon_{xyH}$  and  $E_{av}$ . For  $\alpha_j = \{0.6, 0.7, 1.0\}$  the values of  $\varepsilon_{xyH}$  and  $E_{av}$  are slightly higher comparatively to  $\alpha_j = 0.5$ . For  $\alpha_j = 0.4$  both  $\varepsilon_{xyH}$  and  $E_{av}$  present much higher values. In the case of  $\alpha_j = 0.8$  the robot locomotion presents lower values of  $\varepsilon_{xyH}$  but requires higher values of  $E_{av}$ .

For values of  $\alpha_j = \{0.1, 0.2, 0.3, 0.4\}$ , the results are very poor and for  $-0.9 < \alpha_j < -0.1$  and  $\alpha_j = 0.9$ , the hexapod locomotion resulted unstable. Furthermore, comparing Figures 4 and 5, we conclude that the best case corresponds to all leg joints being motor actuated. This can also be concluded through the observation of Figures 6 and 7 that present the plots of the leg joint actuation torques  $\tau_{ljm}$  and of the hip trajectory tracking errors  $\Delta_{lxH}$  and  $\Delta_{lyH}$  versus  $t$  for the cases of having the ankle joint mechanical actuated and motor actuated, considering the controller fractional order  $\alpha_j = 0.5$ .

In order to fully understand the differences between the FO  $PD^\alpha$  and the classical PD controller tuning we further compare the joint actuation torques  $\tau_{ljm}$  and the hip trajectory tracking errors  $\Delta_{lxH}$ , for the case of all joints motor actuated.

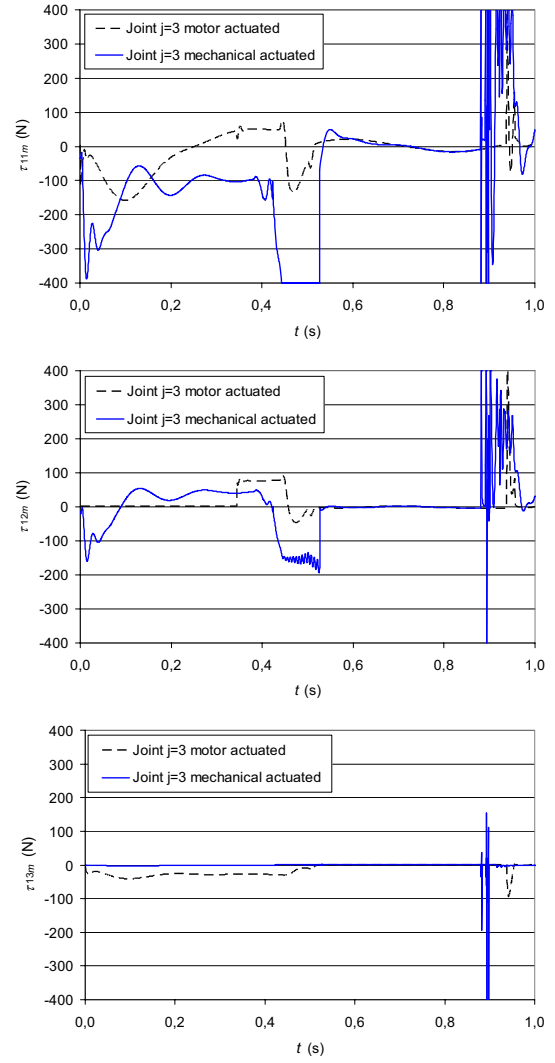


Fig. 6. Plots of  $\tau_{ljm}$  vs.  $t$ , with joints 1 and 2 motor actuated and joint 3 mechanical actuated and all joints motor actuated, for  $\alpha_j = 0.5$ .

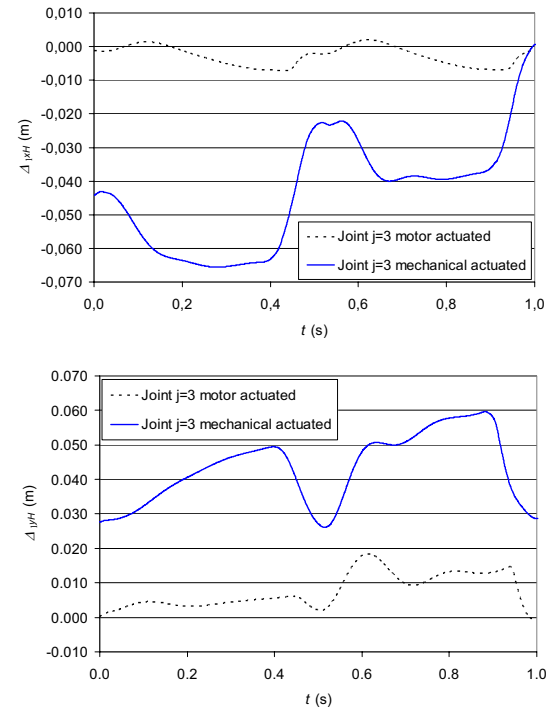


Fig. 7. Plots of  $\Delta_{lxH}$  and  $\Delta_{lyH}$  vs.  $t$ , with joints 1 and 2 motor actuated and joint 3 mechanical actuated and all joints motor actuated, for  $\alpha_j = 0.5$ .



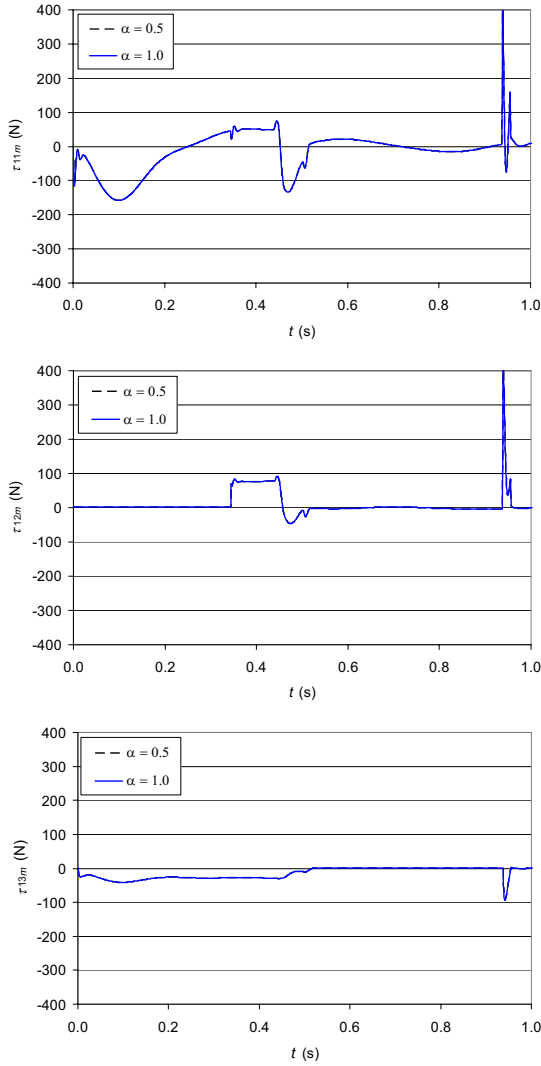


Fig. 8. Plots of  $\tau_{1jm}$  vs.  $t$ , with all joints motor actuated, for the FO PD $^\alpha$  ( $\alpha_j = 0.5$ ) and the classical PD ( $\alpha_j = 1.0$ ) controller implementations

We conclude that both controller tunings present similar curves for the joint actuation torques  $\tau_{1jm}$  (Figure 8) and for the hip trajectory tracking errors  $\Delta_{1xH}$  and  $\Delta_{1yH}$ .

Since the objective of the walking robots is to walk in natural terrains, in the sequel we test how the different controllers behave under distinct ground properties. We conclude that the controller responses are quite similar, meaning that these algorithms are robust to variations of the ground characteristics (Silva and Machado, 2006b).

It is worth mentioning that in the case when joint 3 is mechanically actuated, the robot puts the toe tips in the ground, followed by the ankle. Both stay in this state during the feet support phase and, consequently, the robot walks supporting its body in link  $L_{i3}$ . On the contrary, when all joints are motor actuated, during the feet support phase, the robot walks in its toe tips. By other words, the hexapod supports itself in the extremity of link  $L_{i3}$ .

From the biological point of view both cases are important. Therefore, further study is necessary to understand more deeply how the behaviour change with the locomotion parameters.

## 6. CONCLUSIONS

In this paper we have compared the performance of different FO robot controller for joint leg control of an hexapod robot, both for the mechanical and motor actuated ankle joint.

In order to analyze the system performance two measures were defined based on the mean absolute density of energy per travelled distance and the hip trajectory errors. The experiments reveal the superior performance of the FO controller for  $\alpha_j \approx 0.5$  and a robot with all motor actuated joints.

The focus of the work presented has been on FO PD $^\alpha$  controllers with a proportional plus a derivative/integrative term. Presently we are studying the performance of the system in case we have a FO PID control algorithm of the type PI $^2$ D $^\alpha$ . Future work in this area will also address the study of the performance of these controllers when the hexapod is faced with further variable ground conditions, obstacles and different locomotion parameters.

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