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# Centralized and Decentralized Applications of a Novel Adaptive Control

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**Abstract** – An adaptive control based on the combination of a novel branch of Soft Computing and fractional order derivatives was applied to control two incompletely modeled, nonlinear, coupled dynamic systems. Each of them contained one internal degree of freedom neither directly modeled/observed nor actuated. As alternatives the decentralized and the centralized control approaches were considered. In each case, as a starting point, a simple, incomplete dynamic model predicting the state-propagation of the modeled axes was applied. In the centralized approach this model contained all the observable and controllable joints. In the decentralized approach two similar initial models were applied for the two coupled subsystems separately. The controllers were restricted to the observation of the generalized coordinates modeled by them. It was expected that both approaches had to be efficient and successful. Simulation examples are presented for the control of two double pendulum-cart systems coupled by a spring and two bumpers modeled by a quasi-singular potential. It was found that both approaches were able to “learn” and to manage this control task with a very similar efficiency. In both cases the application of near integer order derivatives means serious factor of stabilization and elimination of undesirable fluctuations. Since in many technical fields the application of simple decentralized controllers is desirable the present approach seems to be promising and deserves further attention and research.

## I INTRODUCTION

In the modern approaches of control technology the use of uniform mathematical structures and forms is a strengthening trend. For instance, an important class of physical systems’ control is the set of non-stationary stochastic processes in which some deterministic response to an external input and a stationary stochastic process are superimposed. This is relevant, for instance, when the external input cannot be effectively described by some probabilistic distribution. A discrete time model can be formulated in the form of a difference equation with an external input  $\{u_k\}$  that is usually considered to be known (Autoregressive Moving Average Model with external input - ARMAX) [1]:

$$y_{k+1} = \sum_{s=0}^N a_s y_{k-s} + \sum_{w=0}^M b_w u_{k-w} \quad (1)$$

In the so-called Takagi-Sugeno fuzzy models the consequent parts are expressed by analytical expressions similar to (1). The TS fuzzy controllers use some linear combinations of the (1)-type rules in which the coefficients depend on the antecedents. With the help of such Takagi-Sugeno fuzzy IF-THEN rules sufficient conditions to check the stability of fuzzy control systems are now available. These schemes are based on the stability theory of interval matrices and those of the Lyapunov approach

[2]. It was already observed that the fuzzy controller stability conditions can be rewritten in form of Linear Matrix Inequalities (LMIs) [3, 4]. LMIs can be efficiently solved numerically by solving very complex Riccati equations for a positive definite solution [5].

Neural Networks in general are useful means of developing nonlinear models. A particular case of such applications is when the model itself consists of certain nonlinear mapping, for instance in the linearization of the nonlinear characteristics of various sensors [6]. Neuro-fuzzy systems provide the fuzzy systems with automatic tuning systems using Neural Network (NN) as a tool. The Adaptive Neuro-Fuzzy Inference System (ANFIS) is a cross between an artificial neural network and a Fuzzy Inference System (FIS) [2, 7, 8, 9]. The adaptive network can be a multi-layer feed-forward network in which each node (neuron) performs a particular function on incoming signals. Based on the ability of an ANFIS to learn from training data, it is possible to create an ANFIS structure from an extremely limited mathematical representation of the system. The ANFIS system generated by the fuzzy toolbox available in MATLAB allows the generation of a standard Sugeno style fuzzy inference system or a fuzzy inference system based on sub-clustering of the data [10]. Radial Basis Function Networks (RBFNs) provide an attractive alternative to the standard Feedforward Networks using backpropagation learning technique [11]. The linear weights associated with the output layer can be treated separately from the hidden layer neurons. As the hidden layer weights are adjusted through a nonlinear optimization, output layer weights are adjusted through linear optimization [2]. In fact the nodes of a RBFN represent “fuzzified” or “blurred” regions which correspond to the well defined antecedent sets of a fuzzy controller. The neuron’s firing achieves its maximum at the centre of the region while its strength decreases with the distance from the center according to some Gaussian function (various distance measures can also be used). In many cases development of the whole model is a complicated task especially when the “antecedent” part is strongly nonlinear multivariable function of the input. Evolutionary methods as e.g. the Particle Swarm Optimization Method that realizes stochastic random search in a multi-dimensional optimization space [12, 13] therefore may also be combined with them. In the case of certain problem classes similarity relations can also be observed and utilized to simplify the design process [14].

A significant common feature of the above approaches is that they try to develop a “complete” soft computing based model of the system to be controlled. This naturally makes the question arise whether it is always reasonable to try to identify a “complete” model. As a plausible alternative

simple adaptive controllers can be imagined that do not wish to create a complete model. Instead of that on the basis of slowly fading recent information a more or less temporal model can be constructed and updated step by step by the use of simple updating rules consisting of finite algebraic steps of lucid geometric interpretation. Realizing that "generality" and "uniformity" of the "traditional SC structures" excludes the application of plausible simplifications made the idea rise that by addressing narrower problem classes a novel branch of soft computing could be developed by the use of far simpler and far more lucid uniform structures and procedures than the classical ones. The first steps in this direction were made in the field of Classical Mechanical Systems (CMSs) [15], based on the Hamiltonian formalism detailed e.g. in [16]. This approach used the internal symmetry of CMSs, the Symplectic Group (SG) of Symplectic Geometry in the tangent space of the physical states of the system. The "result" of the "situation-dependent system identification" was a symplectic matrix compensating the effects of the inaccuracy of the rough dynamic model initially used as well as the external dynamic interactions not modeled by the controller. By the use of perturbation calculus it was proved that under certain restrictions this new approach could be successful in the control of the whole class of classical mechanical systems [17]. (It is interesting that the method of Taylor series extension combined with the Hamiltonian formalism is widely used in our days for problem solution, e.g. [18, 19].) Later it became clear that all the essential steps used in the control could be realized by other mathematical means than the symplectic matrices related to some phenomenological interpretation. Other Lie groups defined in similar manner by some basic quadratic expression like in the case of the Generalized Lorentz Group [20], or symplectic matrices of special structure [21]. The main advantage of using such groups in comparison with the ARMAX-based observations is that while the latter may result in singular or badly conditioned system model to be used for the prediction, the Lie group based models are never singular. (Of course, this fact itself cannot evade all the possible numerical problems.) In comparison with the other Soft Computing methods the use of simple, small uniform *a priori* known size can be mentioned.

Another important aspect in connection with incomplete modeling is the existence of two possible alternative approaches: application of a single, complex rough initial model containing each modeled degree of freedom, or tackling the problem in a "decentralized" manner in which certain subsystems are controlled by independent controllers modeling and controlling only certain degrees of freedom of the subsystem in their care. In this case, for the local, decentralized controllers, any dynamic coupling between the locally controlled subsystems appears as external perturbation influencing the behavior of the subsystem under their control. This problem was discussed in details e.g. in a plenary speech by D'Andrea in connection with the dynamic coupling of wings located in each other's vicinity in flowing air [22]. Since the novel soft computing approach offers simple and convenient implementation for both approaches, and according to the former investigations it was found to be able to manage the consequences of dynamic coupling with unmodeled and

uncontrolled subsystems, it was expedient to investigate its operation in "decentralized use" and comparing the so obtained results with that of the "centralized use". In the sequel at first the paradigm is set mathematically, and following that the basic principles of the adaptive control is described. Following the presentation of the typical simulation results the conclusions are drawn.

## II THE DYNAMIC MODEL OF THE COUPLED SUBSYSTEMS

The cart under consideration consisted of a body and wheels of negligible momentum and inertia having the overall mass of  $M$  [kg]. The pendulums were assembled on the cart by parallel shafts and arms of negligible masses and lengths  $L_1$  and  $L_2$  [m], respectively. At the end of each arm a ball of negligible size and considerable mass ( $m_1$  and  $m_2$ ) [kg] were attached, respectively. The Euler-Lagrange equations of motion of this system are given as follows:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \mathbf{M} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} -m_1 L_1 \cos q_1 \dot{q}_1 \dot{q}_3 - m_1 g L_1 \cos q_1 \\ -m_2 L_2 \cos q_2 \dot{q}_2 \dot{q}_3 - m_2 g L_2 \cos q_2 \\ -m_1 L_1 \cos q_1 \dot{q}_1^2 - m_2 L_2 \cos q_2 \dot{q}_2^2 \end{bmatrix} \quad (2)$$

in which  $\mathbf{M}$  denotes the inertia matrix

$$\mathbf{M} = \begin{bmatrix} m_1 L_1^2 & 0 & -m_1 L_1 \sin q_1 \\ 0 & m_2 L_2^2 & -m_2 L_2 \sin q_2 \\ -m_1 L_1 \sin q_1 & -m_2 L_2 \sin q_2 & (M + m_1 + m_2) \end{bmatrix} \quad (3)$$

In the above formulae  $g$  denotes the gravitational acceleration [ $m/s^2$ ],  $Q_1$  and  $Q_2$  [ $N \times m$ ] denote the driving torque at shaft 1 and 2, respectively, and  $Q_3$  [ $N$ ] stands for the force moving the cart in the horizontal direction. The appropriate rotational angles are  $q_1$  and  $q_2$  [rad], and the linear degree of freedom belongs to  $q_3$  [m]. The 1<sup>st</sup> rotational and the linear degrees of freedom were the controlled and actuated ones, while the second rotary axis is without observation, control, and actuation that means that  $Q_2$  took the constant value zero. Furthermore, two pieces of the above described subsystems were coupled along their linear direction of motion by the forces  $Q_3^A = -Q_3^B$  given in [ $N$ ] as

$$Q_3^A = k(q_3^B - q_3^A - L_0) + \frac{A}{(\varepsilon_{bump} + q_3^B - q_3^A - 1.5 \times L_0)^2} - \frac{A}{(\varepsilon_{bump} + q_3^B - q_3^A - 0.5 \times L_0)^2} \quad (4)$$

in which  $k$  describes a spring stiffness in [ $N/m$ ] units, and  $L_0$  [m] belongs to the zero spring force separation. To model the buffers two non-linear terms are applied that are very sharp near the  $0.5 \times L_0$  and  $1.5 \times L_0$  separations, while in the "internal points" it is very flat. It is described by two parameters, namely by the "strength"  $A$  [ $N \times m^2$ ], and a small parameter  $\varepsilon_{bump}$  [m] determining the "nearness" of the singularity of these coupling forces. In the sequel the principles of the adaptive control are detailed.

## III THE ADAPTIVE CONTROL

From mathematical point of view the can be formulated as follows. There is given some imperfect model of the system on the basis of which some excitation is calculated

to obtain a desired system response  $\mathbf{i}^d$  as  $\mathbf{e}=\boldsymbol{\varphi}(\mathbf{i}^d)$ . The system has its inverse dynamics described by the unknown function  $\mathbf{i}^r=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d))=\mathbf{f}(\mathbf{i}^d)$  and resulting in a realized response  $\mathbf{i}^r$  instead of the desired one,  $\mathbf{i}^d$ . Normally one can obtain information via observation only on the function  $\mathbf{f}()$  considerably varying in time, and no any possibility exists to directly "manipulate" the nature of this function: only  $\mathbf{i}^d$  as the input of  $\mathbf{f}()$  can be "deformed" to  $\mathbf{i}^{d*}$  to achieve and maintain the  $\mathbf{i}^d=\mathbf{f}(\mathbf{i}^{d*})$  state. The following "scaling iteration" was suggested for finding the proper deformation:

$$\begin{aligned} \mathbf{i}_0; \mathbf{S}_1 \mathbf{f}(\mathbf{i}_0) = \mathbf{i}_0; \mathbf{i}_1 = \mathbf{S}_1 \mathbf{i}_0; \dots; \mathbf{S}_n \mathbf{f}(\mathbf{i}_{n-1}) = \mathbf{i}_0; \\ \mathbf{i}_{n+1} = \mathbf{S}_{n+1} \mathbf{i}_n; \mathbf{S}_n \xrightarrow{n \rightarrow \infty} \mathbf{I} \end{aligned} \quad (5)$$

in which the  $\mathbf{S}_n$  matrices denote some linear transformations to be specified later. As it can be seen these matrices maps the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, therefore the controller „learns“ the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Details of ambiguity resolution of (5) and finding the proper  $\mathbf{S}_n$  matrices on group-theoretical basis were published in many times. Regarding the appropriate details we refer to [20, 21]. In the sequel the significance of the application of fractional order derivatives is emphasized. Since according to (2) in the role of the “response” the 2<sup>nd</sup> order time-derivatives, while in the role of the “excitation” the generalized coordinates as joint forces and torques are in the case of a mechanical system, on the basis of purely kinematical considerations prescribing a PID-type error-relaxation, and by the use of a rough dynamic model consisting of a constant scalar inertia matrix  $M_m$ , and a constant additional vector term  $\mathbf{b}$ , the generalized forces can be estimated as.

$$\begin{aligned} \mathbf{Q} = M_m \ddot{\mathbf{q}}^D + \mathbf{b}, \\ \ddot{\mathbf{q}}^D(t) = \ddot{\mathbf{q}}^N(t) + D(\dot{\mathbf{q}}^N(t) - \dot{\mathbf{q}}^R(t)) + \\ + P(\mathbf{q}^N(t) - \mathbf{q}^R(t)) + I \int_{t_0}^t (\mathbf{q}^N(\tau) - \mathbf{q}^R(\tau)) d\tau \end{aligned} \quad (6)$$

By the use of Caputo's definition of fractional order derivatives

$$\frac{d^\beta}{dt^\beta} u(t) := \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{du(\tau)}{d\tau} (t-\tau)^{-\beta} d\tau, \beta \in (0,1) \quad (7)$$

(6) can be modified as  $\mathbf{Q} = M_m \mathbf{q}^{D(1+\beta)} + \mathbf{b}$ , in which the desired  $(1+\beta)^{th}$  order derivative is calculated by replacing the 2<sup>nd</sup> order desired derivative into (7). In this solution (7) can be regarded as a *temporal filtered average* of the integer order 2<sup>nd</sup> derivative from which the noisy fluctuations of the desired 2<sup>nd</sup> derivative are “integrated out”. In the  $t>0$  region the “tail” of the  $(t-\tau)^{-\beta}$  kernel function really acts as a frequency filter rejecting the high frequency fluctuations, while its singularity in  $t=\tau$  enhances the relatively high significance of the actual time. Observing the fact that for constant  $du/dt$  (7) can analytically calculated, one of its practical numerical

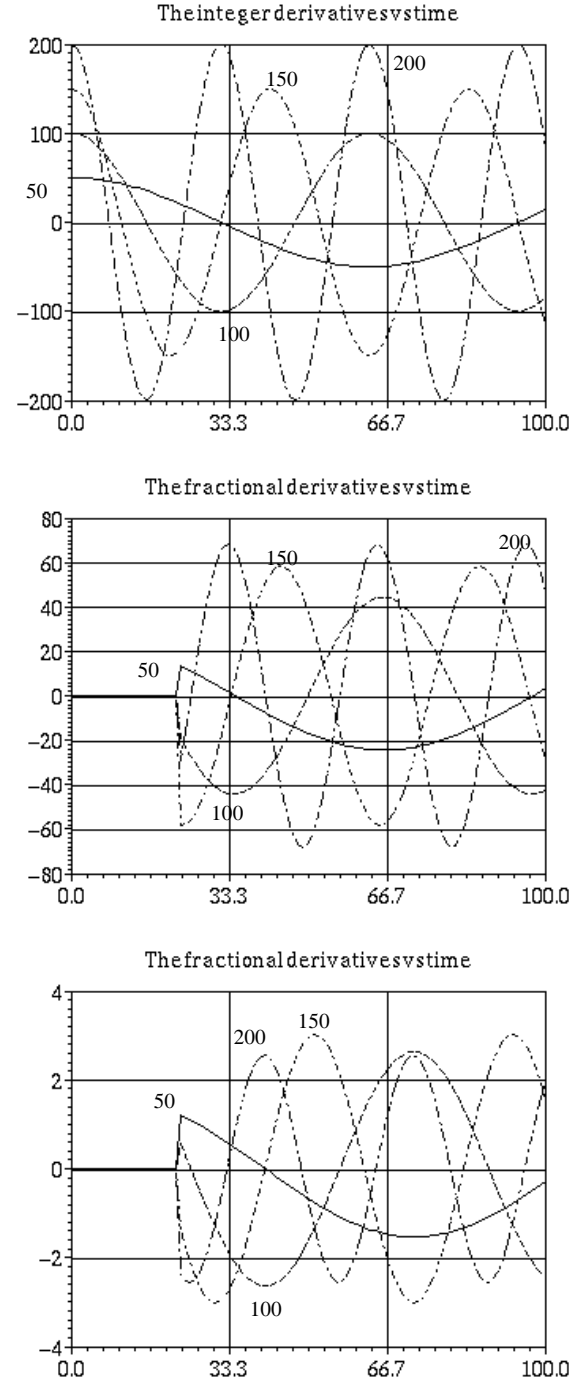


Figure 1  
The  $\beta=1$  (top), 0.8 (middle), and 0.1 (bottom) order derivatives of a sinusoidal signal of unit amplitude of circular frequencies 50, 100, 150, and 200 Hz

approximations can be obtained by restricting the system's memory to a final  $[t, t-T]$  interval, dividing it into small subintervals along which the variation of  $du/dt$  is neglected:

$$\begin{aligned} \frac{d^\beta}{dt^\beta} u(t) \cong \frac{u'(t) \delta^{-\beta+1}}{\Gamma(2-\beta)} + \\ + \sum_{0 < s \leq t \text{ while } s\delta < T} \frac{\delta^{-\beta+1} [(s+1)^{-\beta+1} - s^{-\beta+1}]}{\Gamma(2-\beta)} u'(t-s\delta) \end{aligned} \quad (8)$$

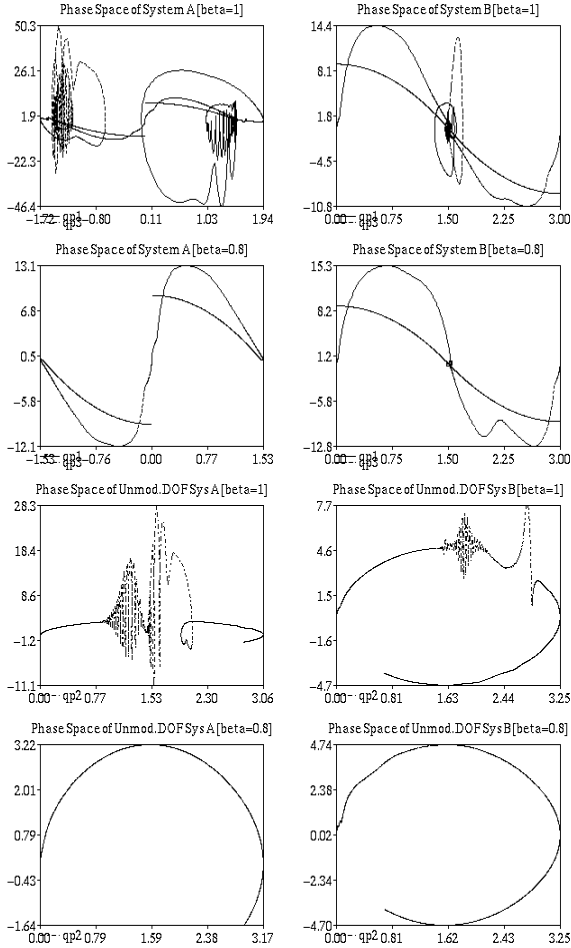


Figure 2

Typical operation of the adaptive *decentralized control*: the phase spaces of the controlled subsystems  $\{[m/s] \text{ vs. } [m] \text{ and } [rad/s] \text{ vs. } [rad]\}$  for  $\beta=1$  (1st row) and for  $\beta=0.8$  (2nd row), and the phase-spaces of the uncontrolled axes  $\{[rad/s] \text{ vs. } [rad]\}$  for  $\beta=1$  (3rd row) and for  $\beta=0.8$  (4th row).

It is worth noting that (8) exactly yields the integer 1<sup>st</sup> derivative as  $\beta \rightarrow 1$ . The effects of fractional order derivation can well be illustrated via calculating (8) for  $\delta=1 \text{ ms}$  long intervals of division and  $T=20 \text{ ms}$  long “memory” in the case of a sinusoidal signal of unit amplitude and circular frequency of 50, 100, 150, and 200 Hz: a) with decreasing order decreases the amplitudes of the derivatives; b) the higher frequencies are rather suppressed than the lower ones; c) some phase-ships can be observed that increases with the order of derivation. In the adaptive control the appropriate fractional order derivatives are compared to each other on the construction of the necessary “deformation”. In the next part simulation examples are given for  $\delta=1 \text{ ms}$ ,  $T=10 \text{ ms}$ , and  $\beta=1$  and 0.8.

#### IV SIMULATION RESULTS

In the simulations for the desired relaxation of the trajectory tracking error a simple PID-type rule was prescribed by the use of purely kinematic terms. This error relaxation could be achieved exactly only in the possession of the exact dynamic model of the system to be controlled. Subsystem A had the following numerical data:  $m_1^A=10 \text{ kg}$ ,  $m_2^A=10 \text{ kg}$ ,  $L_1^A=2 \text{ m}$ ,  $L_2^A=2 \text{ m}$ ,  $M_A=4 \text{ kg}$ . In order to introduce asymmetry into the system subsystem B had the following data:  $m_1^B=20 \text{ kg}$ ,  $m_2^B=10 \text{ kg}$ ,  $L_1^B=3 \text{ m}$ ,  $L_2^B=1 \text{ m}$ ,

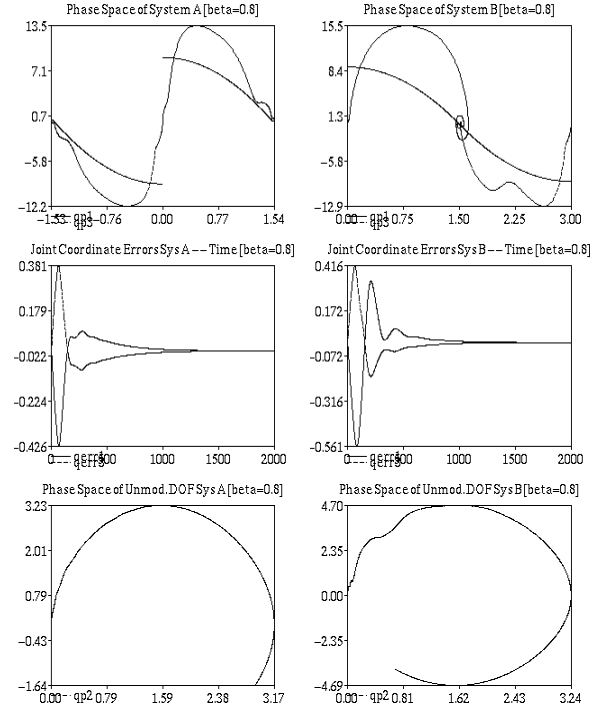


Figure 3

Typical operation of the adaptive *centralized control* for  $\beta=0.8$ : the phase spaces  $\{[m/s] \text{ vs. } [m] \text{ and } [rad/s] \text{ vs. } [rad]\}$  (1st row) and the trajectory tracking error  $[m, rad] \text{ vs. time } [ms]$  (2nd row) of the controlled subsystems; phase-spaces of the uncontrolled axes  $\{[rad/s] \text{ vs. } [rad]\}$  (3rd row);

$M_B=6 \text{ kg}$ . The coupling spring had the stiffness of  $k=10^4 \text{ N/m}$ , the “bumper’s force constant” was  $10^3 \text{ N} \times m^2$ , and  $\varepsilon_{bump}=10^{-3} \text{ m}$ . The separation belonging to the zero spring force was  $L_0=3 \text{ m}$ . Instead of the exact actual dynamic model the constant diagonal inertia matrix containing the elements  $10 \text{ [kg} \times m^2]$  or  $[kg]$  in its main diagonal, and having the numerical value 10 in the matrix elements in the role of the sum of the gravitational and Coriolis terms was used in both the centralized and the decentralize cases. (Only the sizes of the appropriate arrays were different to each other.) The cycle-time of the controller was supposed to be  $1 \text{ [ms]}$ .

In Fig. 2 typical results can be seen for the *adaptive decentralized control*. In the  $\beta=1$  case certain instability can be observed that successfully is eliminated by a little decrease of the order of time-differentiation applied ( $\beta=0.8$ ). Fig. 3 displays the counterpart of this latter case for the *adaptive centralized control*. The numerical results of the centralized and decentralized approaches are very close to each other, minor differences in the operation can be observed only in the relaxation of the rough initial transient of the controlled axes, mainly in the phase space: each of them has zero initial velocity while the “nominal trajectory” starts with considerable one. The nominal trajectories in each case asymptotically approach the zero velocity as well as the computed ones.

The significance of the adaptivity is illustrated by Fig. 4 that is the non-adaptive counterpart of the control illustrated by Fig. 3. Continuous use of the rough initial dynamic models leads to unacceptable results.

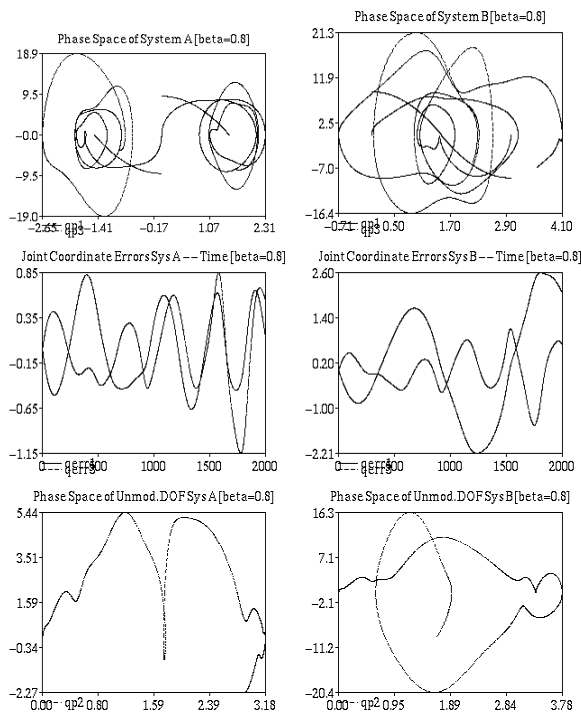


Figure 4

Non-adaptive counterpart of the *centralized control* (Fig. 3) for  $\beta=0.8$ : the phase spaces  $\{[m/s] \text{ vs. } [m] \text{ and } [rad/s] \text{ vs. } [rad]\}$  (1st row) and the trajectory tracking error  $[m, rad] \text{ vs. time } [ms]$  (2nd row) of the controlled subsystems; phase-spaces of the uncontrolled axes  $\{[rad/s] \text{ vs. } [rad]\}$  (3rd row);

## V CONCLUSIONS

In this paper the behavior of the *decentralized* and the *centralized* application of an adaptive control method based on a novel branch of Computational Cybernetics and fractional time-derivatives were compared to each other. In the paradigm investigated the approximately modeled, coupled non-linear subsystems also had unmodeled and uncontrolled internal degrees of freedom. The simulation results well illustrated that both ways of the application of adaptivity considerably improved the quality of the trajectory reproduction and successfully compensated the effects of coupling between the subsystems. The application of a near integer derivative in the kinematically prescribed trajectory tracking strategy well smoothed and stabilized the operation of the controllers. The results anticipate that this novel method can be a useful means for a practically advantageous decentralized control of various coupled, incompletely and inaccurately modeled subsystems.

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