Buckling strength of adhesively-bonded single and double-strap repair on carbon-epoxy structures


ABSTRACT

This work reports on an experimental and finite element method (FEM) parametric study for adhesively- bonded single and double-strap repairs on carbon-epoxy structures under buckling unrestrained compression. The influence of the overlap length and patch thickness was evaluated. This loading gains a particular significance from the additional characteristic mechanisms of structures under compression, such as fibres microbuckling, for buckling restrained structures, or global buckling of the assembly, if no transverse restriction exists. The FEM analysis is based on the use of cohesive elements including mixed-mode criteria to simulate a cohesive fracture of the adhesive layer. Trapezoidal laws in pure modes I and II were used to account for the ductility of most structural adhesives. These laws were estimated for the adhesive used from double cantilever beam (DCB) and end-notched flexure (ENF) tests, respectively, using an inverse technique. The pure mode III cohesive law was equalled to the pure mode II one. Compression failure in the laminates was predicted using a stress-based criterion. The accurate FEM predictions open a good prospect for the reduction of the extensive experimentation in the design of carbon-epoxy repairs. Design principles were also established for these repairs under buckling.

Keywords: Laminate, Carbon fibres, Fracture, Finite element analysis (FEA) Cohesive zone model

1. Introduction

Carbon-fibre reinforced plastics (CFRP) have recently come to the fore owing to a set of interesting characteristics over conventional materials such as aluminium or steel. In fact, CFRP composites are being increasingly used in structures requiring high specific strength and stiffness, such as in the automotive, marine, military, aeronautic and aerospace industries. The replacement costs of damaged CFRP components highlight the importance on the availability of effective repair procedures to restore their strength. Adhesively-bonded repairs overcome the limitations of the riveting or fastening approaches, such as the weight penalty or significant stress concentrations at localized regions, which can induce premature onset of damage. However, bonded assembly blies are more likely to fail under moderate fatigue stress levels [1,2]. The strap repair consists on drilling a hole to remove the damaged material, followed by adhesive-bonding of circular patches on one side (single strap, SS) or both sides (double strap, DS) of the laminate [3]. These repairs can be regarded as permanent in lightly loaded and relatively thin structures. However, they are not feasible for high responsibility structures, due to peel and shear stress concentrations at the bond edges, arising from the differential straining between the structure and patches along the bond length [3], which prevents a high efficiency.

Several authors addressed adhesively-bonded repair techniques for composite structures [4–14]. Numerically, the repairs’ strength can be predicted using stress or strain-based criteria for the adhesive layer, damaged structure and patch [15–20]. However, because of the stress concentrations or singular regions typical of these repairs, the mentioned criteria are highly mesh dependent. Alternatively, cohesive zone models (CZM’s) coupled to FEM simulations can be used [3,14,21–23], modelling damage growth in the adhesive layer and, in the case of layered fibre-reinforced composites, also interlaminar, intralaminar or fibre fractures. This methodology yields mesh independent results, since damage growth is ruled by energetic criteria. Additionally, accurate failure mechanisms predictions can be expected, provided that all fractures are considered in the simulations. The DS repair technique on CFRP laminates under tension loads was studied by Liu and Wang [24]. An optimization study was carried out for the patch diameter, thickness and lay-up, as well as the adhesive thickness. The FEM simulations precisely estimated the repairs experimental strength, using the Tsai-Wu criterion to predict fibre and matrix...
cracking of the laminate and patches, the Ye delamination criterion to detect delamination between plies and the Maximum Shear Stress Criterion for the adhesive layer. The compressive behaviour of composite structures must also be considered in the design stages. In fact, different mechanisms arise under compression, such as fibres microbuckling, for buckling of restrained structures [25–28], or global buckling of the assembly between clamping points, if no transverse restriction exists [29]. Fibres microbuckling comes from the compliant materials used as matrix, whilst global buckling is justified by the slenderness of elements, largely depending on their crosssection and elastic properties, in addition to the imposed boundary and loading conditions. When loaded on its length direction, a slender member under unrestrained compression initially deforms in pure compression. At a certain level of applied load, elastic instability of the beam leads to its transverse deflection accompanied by crosssectional rotation around the axis of smaller second moment of inertia, which grows at an approximate constant load. This phenomenon is known as lateral buckling, and the corresponding load as the buckling load [29]. Owing to these mechanisms, fibre-reinforced laminates sustain smaller loads in compression than tension [30,31], implying that composite structures under bending are more likely to fail in the compression region [32]. The accurate prediction of the stability limit states of CFRP repairs thus shows a particular importance, owing to the mentioned phenomena. This justified the attention paid to the development of theoretical and computational methods for the buckling analysis of slender elements, with emphasis on thin-walled composite panels [33,34]. With this purpose, analytical approaches based on the Vlasov theory were applied to conventional [35] and composite structures [36,37]. A few closed-form solutions were also developed for the buckling loads of slender structures [38–41]. Soutis et al. [42] performed a FEM analysis on the influence of several geometric parameters on the compressive strength of DS composite repairs, using a designed apparatus to prevent lateral buckling. The strength and locus of damage initiation of the repairs were estimated with the Maximum Normal Stress Criterion for the microbuckling in the laminate and the Average Shear Stress Criterion for the adhesive layer failure, giving accurate predictions. The work of Finn et al. [43] concerns the behaviour of composite repairs under compression. The authors experimentally estimated the effectiveness of strap repairs on the compressive strength of CFRP laminates without global buckling restraining. It was found that the applied load increased until the buckling load, representing the load at which the global buckling between grips initiated. A plateau region followed in the load–displacement (P–d) curve, corresponding to buckling, up to failure by extensive delamination. Results indicated that for the damaged laminates used in the tests, no significant improvement of the compressive strength was attained. Therefore, the recommended course of action was not to execute the repair. This work reports on an experimental and FEM parametric study of adhesively-bonded SS and DS repairs on CFRP laminates with a [0°,90°,0°,90°]s lay-up under buckling unrestrained compression. The influence of the overlap length (L0) and patch thickness (t0) was evaluated, which allowed the establishment of design principles for repairing of CFRP structures. The FEM analysis is based on the use of cohesive elements including mixed-mode criteria to simulate a cohesive fracture of the adhesive layer. Trapezoidal traction-separation laws in pure modes I and II were considered to reproduce the ductile characteristics of most struc-

to the pure mode II one. Compression failure in the laminate emerging from the specimens buckling was predicted using a stress-based criterion.

2. Cohesive zone model

2.1. Model description

A mixed-mode (I + II + III) CZM implemented within cohesive elements was used to simulate the elasto-plastic behaviour and fracture of an adhesive layer of Araldite 2015 with thickness (tA) of 0.2 mm. The cohesive elements are used in the numerical models to connect the solid elements of ABAQUS simulating the laminate and patch, to reproduce the behaviour of the adhesive layer by using approximate traction-separation laws in each mode of loading. A trapezoidal law was considered for each pure mode to relate stresses (σr) and relative displacements (dr) between homologous points of the cohesive elements with zero thickness (Fig. 1), to account for the adhesive ductility [44–46]. Actually, these adhesives are characterized by an approximate constant stress under plastic deformation, which can be approximated to the cohesive shape of Fig. 1. The homologous points of the cohesive elements correspond to the initially superimposed points of the cohesive elements in the numerical model, which are connected by the mixed-mode CZM of Fig. 1, following the established r–d relationship up to failure. In general, structures are under mixed-mode. Therefore, the formulation includes a mixed-mode damage model, which is an extension of the pure-mode model (Fig. 1) and allows the simulation of the adhesive layer when under mixed-load conditions. Damage onset is predicted using the quadratic stress criterion

\[
\left( \frac{\sigma_{r1}}{\sigma_{r1}^c} \right)^2 + \left( \frac{\sigma_{r2}}{\sigma_{r2}^c} \right)^2 + \left( \frac{\sigma_{r3}}{\sigma_{r3}^c} \right)^2 = 1 \quad \text{if } \sigma_{r1} > 0
\]

\[
\left( \frac{\sigma_{r1}}{\sigma_{r1}^c} \right)^2 + \left( \frac{\sigma_{r2}}{\sigma_{r2}^c} \right)^2 = 1 \quad \text{if } \sigma_{r1} < 0
\]

where \( \sigma_r (i = I, II, III) \) represent the stresses in each mode and \( \sigma_{r1} (I, II, III) \) the corresponding local strength. The first Eq. (1) can be rewritten as a function of the relative displacements

\[
\left( \frac{\delta_{r1}}{\delta_{r1}^c} \right)^2 + \left( \frac{\delta_{r2}}{\delta_{r2}^c} \right)^2 + \left( \frac{\delta_{r3}}{\delta_{r3}^c} \right)^2 = 1.
\]

dr (i = I, II, III) represent the pure mode relative displacements at damage initiation and dr (i = I, II, III) the corresponding mixed-mode displacements. Stress softening onset was predicted using a criterion similar to (2)
tural adhesives, such as the epoxy adhesive Araldite 2015 used in this work. The cohesive laws in pure modes I and II were estimated from DCB and ENF tests, respectively, using an inverse data fitting methodology. The pure mode III cohesive law was equalled
As a result, failure extends tough engineering adhesives, height and non-the adhesive bulk properties. Cohesive parameters in pure modes I and II were estimated by an adhesives layer in pure modes I and II were estimated by an 2.2. Cohesive parameter fitting. The trapezoidal mixed-mode CZM presented in the previous section was used in the numerical models to simulate a t_A = 0.2 mm layer of Araldite. The adhesive layer elastic stiffness in tension and shear (up to d_{i,n}, Fig. 1) is defined from the experimentally measured values of Young’s modulus (E = 1850 MPa) and shear modulus (G = 650 MPa) [48], as detailed in the work of Campilho et al. [47]. In the present work, the cohesive laws of the adhesive layer in pure modes I and II were estimated by an inverse data fitting procedure on DCB (mode I) and ENF (mode II) tests, whilst the pure mode III cohesive law was equalled to the pure mode II one, as an approximation (Table 1). A detailed description of the inverse technique can be found in the works of de Moura et al. [49–51]. This course of action is supported by the typically varying mechanical properties of thin adhesive layers with t_A, diverging also to the adhesive bulk properties [44]. In fact, the habitually high-modulus and non-yielding adherends in adhe-sive bonds greatly affect the height and extension of the fracture process zone (FPZ). Actually, for tough engineering adhesives, the FPZ height is confined by t_A, but extends much longer ahead of the crack tip than in bulk adhesives. [44–46,52]. Additionally, in bonded assemblies the adhesive layer is typically weaker and more compliant than the components to be joined. As a result, failure is often cohesive in the adhesive layer, which results in a mixed-

\[ \frac{(\frac{d_{i,m}}{d_{i,n}})^3}{d_{i,m}} + \frac{(\frac{d_{i,m}}{d_{i,n}})^3}{d_{i,m}} + \frac{(\frac{d_{i,m}}{d_{i,n}})^3}{d_{i,m}} = 1. \]  

\[ \frac{J_{c} - J_{c,i} - J_{c,II} - J_{c,III}}{J_{c,i}} = 1. \]  

J_{c} (i = I, II, III) is the fracture energy in the respective pure mode. When Eq. (4) is satisfied, damage growth occurs and stresses are completely released, with the exception of normal compressive ones. A detailed description of the proposed model is presented in the work of Campilho et al. [47].

### 2.2. Cohesive parameters

The trapezoidal mixed-mode CZM presented in the previous section was used in the numerical models to simulate a t_A = 0.2 mm layer of Araldite. The adhesive layer elastic stiffness in tension and shear (up to d_{i,n}, Fig. 1) is defined from the experimentally measured values of Young’s modulus (E = 1850 MPa) and shear modulus (G = 650 MPa) [48], as detailed in the work of Campilho et al. [47]. In the present work, the cohesive laws of the adhesive layer in pure modes I and II were estimated by an inverse data fitting procedure on DCB (mode I) and ENF (mode II) tests, whilst the pure mode III cohesive law was equalled to the pure mode II one, as an approximation (Table 1). A detailed description of the inverse technique can be found in the works of de Moura et al. [49–51]. This course of action is supported by the typically varying mechanical properties of thin adhesive layers with t_A, diverging also to the adhesive bulk properties [44]. In fact, the habitually high-modulus and non-yielding adherends in adhesive bonds greatly affect the height and extension of the fracture process zone (FPZ). Actually, for tough engineering adhesives, the FPZ height is confined by t_A, but extends much longer ahead of the crack tip than in bulk adhesives. [44–46,52]. Additionally, in bonded assemblies the adhesive layer is typically weaker and more compliant than the components to be joined. As a result, failure is often cohesive in the adhesive layer, which results in a mixed-

\[ \frac{(\frac{d_{i,m}}{d_{i,n}})^3}{d_{i,m}} + \frac{(\frac{d_{i,m}}{d_{i,n}})^3}{d_{i,m}} + \frac{(\frac{d_{i,m}}{d_{i,n}})^3}{d_{i,m}} = 1. \]  

\[ \frac{J_{c} - J_{c,i} - J_{c,II} - J_{c,III}}{J_{c,i}} = 1. \]  

J_{c} (i = I, II, III) is the fracture energy in the respective pure mode. When Eq. (4) is satisfied, damage growth occurs and stresses are completely released, with the exception of normal compressive ones. A detailed description of the proposed model is presented in the work of Campilho et al. [47].

### 3. Numerical analysis

The strap repair geometry is presented in Fig. 2. The characteristic dimensions of the repair are the laminate length (a) and width (b), the overlap length (L), defined as the radial overlap between the hole and the patch, the laminate thickness (p), the adhesive layer thickness (t_A), the patch thickness (t_B) and the hole diameter (d). It should be mentioned that a denotes the specimens length between grips, while their total length is 150 mm. A parametric analysis was performed, both experimentally and numerically, on the influence of L and t_B on the repairs behaviour, which will allow the establishment of repair principles for these structures. The open-hole specimen (without patches) was also characterized, for an evaluation of the strength improvement for the different repair configurations tested. The buckling behaviour of this notched configuration is extensively documented in the literature, due to its significance, for instance in bolted or riveted structures [27]. Moreover, the need for hole drilling can emerge in composite structures, further emphasizing the importance of the strength evaluation for this notched configuration. It is thus emphasized the particular character of the quantitative predictions presented to the specific set of testing, material and dimensional conditions selected. Table 2 presents the geometries evaluated for the SS and DS repairs, and the patch lay-up for each value of t_B. The [0°,90°,0°,90°] tape-up was always used for the laminates. The label of the repairs defined in this table will be used throughout this work when addressing each one of the repair geometries mentioned, to simplify the nomenclature.

The numerical work was performed in ABAQUS®, using geometric non-linearities due to the large deformations endured by the specimens under buckling, for a faithful representation of their behaviour [54]. Including the non-linear geometric effects is extremely important in the particular case of buckling compression, since a considerable lateral flexure is observed. Under these conditions, a linear geometric analysis would underestimate the bucking behaviour [55,56]. By the use of geometrical non-linearities, finite elements in the numerical simulations are always formulated in the current configuration using current nodal positions, with the update of the stiffness matrix of the structure on every increment. This is essential to guarantee an accurate representation of the real behaviour of the repairs tested, due to their transverse deflection before failure. Eight-node reduced integration solid finite elements were considered for the analysis, to keep the computational effort necessary to run the models at an acceptable level. These elements are compatible with the eight-node cohesive elements simulating the adhesive layer. Hexahedral eight-node and pentahedral six-

![Diagram](image)

<table>
<thead>
<tr>
<th>Pure mode</th>
<th>i</th>
<th>J_{c} (N/mm)</th>
<th>( \Gamma_{\infty} (MPa) )</th>
<th>( d_{i,m} (mm) )</th>
</tr>
</thead>
<tbody>
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<td>I</td>
<td>0.43</td>
<td>23.0</td>
<td>0.01870</td>
</tr>
<tr>
<td>Pure mode</td>
<td>II</td>
<td>4.70</td>
<td>22.8</td>
<td>0.1710</td>
</tr>
</tbody>
</table>

Table 1: Cohesive parameters in pure modes I and II of a t_A = 0.2 mm layer of Araldite 2015.
Fig. 2. SS repair geometry and dimensions.
Node solid finite elements were employed in the simulations (Fig. 3 details the mesh at the repaired region for the L0 = 10 mm SS repair). The use of symmetry conditions at planes A (transversal) and B (longitudinal) allowed the consideration of only 1/4 of the repair (Fig. 4). In all of the geometries, thirty elements were applied along the overlap in the radial direction. The use of bias effects towards the overlap edges guaranteed a refined mesh at these stress concentration regions [57,58]. Thirty elements were also used for l1 of the patch in the circumferential direction. As a result, the cohesive elements for the simulation of the adhesive layer consisted of a layer of 30 x 30 elements. The meshes of the laminate and patch were built using one element through-thickness for each two adjacent equally oriented plies, except near the adhesive, where one element was used per ply because of the expected stress concentrations. The four 90° plies at the middle of the laminate were modelled using only one element through-thickness. Because of this mesh arrangement, the laminates of the SS repairs were modelled by eight plies of elements through-thickness (Fig. 3), whilst nine plies were considered for the laminate of the DS repairs, due to the additional ply of solid elements near the second adhesive layer. Each one of these eight or nine plies consisted of 1500 solid elements. By these principles, the patches were made of 3, 5, 7 or 9 plies of solid elements for the tL1 = 0.6 to the tL1 = 2.4 mm patches, by the respective order, each of these plies consisting of 1200 solid elements. In view of this arrangement, the total number of elements ranged from 16,500 for the tL1 = 0.6 mm SS repair to 36,900 for the tL1 = 2.4 mm DS repair. The CFRP laminates and patches were modelled as orthotropic elastic materials with the following properties for a unidirectional lamina (1-fibres direction, 2-transverse direction, 3-thickness direction): E1 = 109 GPa, E2 = E3 = 8819 MPa, G12 = G13 = 0.342, G23 = 0.380, G11 = G22 = 4315 MPa, G23 = 3200 MPa [3]. As previously mentioned, the cohesive fracture of the adhesive layer was evaluated by cohesive elements. Additionally, since the maximum load (Pmax) for some of the tested conditions was ruled by a compression failure of the two superficial 0° plies at the compression region of the laminates at plane A (Fig. 4), a stress-based criterion was used to predict this occurrence [24,42]. This approach consisted on the experimental estimation of the average value of d leading to compression failure for the open-hole laminates. Following, with the corresponding FEM model, this value of d was used to determine the average value of normal stress in the x direction (tXX) for the two superficial 0° plies at the compression region of the laminate at plane A (Fig. 4). In the simulations of the repairs, the value of tXX was at the mentioned plies was used to assess compression failure. This course of action is substantiated by a practically simultaneous experimental failure at these two plies along the entire width of the specimens, even though it is known that longitudinal compressive stresses along the specimens’ width are not constant, typically concentrating towards the hole edge. This implies that the failure stress obtained by the described inverse procedure cannot be regarded as independent of the repairs geometry, which is a disadvantage. A failure stress to be generally applied to any repair geometry of the same composite material and lay-up can alternatively be obtained using a laminate without a notch, since under these conditions no stress concentrations are present for longitudinal compressive stresses along the width of the specimens.

4. Experimental work

The laminates and patches were fabricated by hand lay-up from unidirectional CFRP pre-preg with 0.15 mm ply thickness (Texi-preg HS 160 RM from SEAIL), followed by curing in a hot-plate press at 130 °C and 2 bar during 1 h. Cutting of the laminates from the bulk plates was achieved by a diamond coated disc saw. The d = 10 mm holes were afterwards executed at in a conventional milling machine using solid carbide Gühring 1149 drills with a 10 mm diameter and an h8 tolerance. These drills were chosen due to their special application to fibre-reinforced materials. The circular patches were manufactured in a conventional turning lathe, using Cubic Boron Nitride inserts from SECO. The patches were also fabricated from bulk plates, initially cut in squares with slightly higher dimensions than the final patch diameter. At this stage, a straight line was marked in the patches, allowing for a sub-sequent correct alignment of the patches with the laminate. Machining of the patches in the turning lathe was carried out be-tween steel cylinders with the same diameter of the patches, firmly tightened between the lathe spindles. The bonding surfaces were abraded with 180 grit sandpaper and cleaned with acetone, to pre-vent adhesive failures [22,59]. To guarantee a uniform value of tX over the entire bond area, a fishing line with a diameter of 0.2 mm was used at four points at the edges between the patches and the laminates. The correct alignment of the patches lay-up was achieved using the straight line previously marked in the patches. To align the patch and certify the concentricity between the patch and the laminate hole, several markings were placed in the laminate. Following, measurements were performed with a digital

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Table 2
Geometries evaluated for the SS and DS repairs.

<table>
<thead>
<tr>
<th>Label (mm)</th>
<th>L0 (mm)</th>
<th>tL1 (mm)</th>
<th>Patch lay-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0 = 5</td>
<td>5</td>
<td>1.2</td>
<td>[0; 90)ls</td>
</tr>
<tr>
<td>L0 = 10</td>
<td>10</td>
<td>1.2</td>
<td>[0; 90)ls</td>
</tr>
<tr>
<td>L0 = 15</td>
<td>15</td>
<td>1.2</td>
<td>[0; 90)ls</td>
</tr>
<tr>
<td>tL = 0.6</td>
<td>10</td>
<td>0.6</td>
<td>[0; 90)ls</td>
</tr>
<tr>
<td>tL = 1.2</td>
<td>10</td>
<td>1.2</td>
<td>[0; 90)ls</td>
</tr>
<tr>
<td>tL = 1.8</td>
<td>10</td>
<td>1.8</td>
<td>[0; 90, 0)ls</td>
</tr>
<tr>
<td>tL = 2.4</td>
<td>10</td>
<td>2.4</td>
<td>[0; 90, 0, 90)ls</td>
</tr>
</tbody>
</table>

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Fig. 3. Detail of the mesh at the overlap region (L0 = 10 mm SS repair).

Fig. 4. Numerical idealization of the strap repairs and boundary conditions.
5. Results

5.1. Fracture mechanisms

Analysis of the results and respective comparison between the experiments and numerical simulations are carried out in this section, beginning with the fracture characterization of the repairs up to failure. The global buckling and damage mechanisms identified during the tests are initially characterized:

- **Global buckling**: Corresponds to buckling between the testing machine grips. Fig. 6a shows an open-hole laminate under global buckling, with the indication of the laminate tension and compression faces. No evidence of damage was found under these conditions up to compression failure.
- **Compression failure**: Damage in the laminate face under compression, being characterized by a compressive fracture of the two superficial 0° plies at plane A (Fig. 4) along the entire specimen width, accompanied by a localized longitudinal matrix cracking at the hole edges [27]. Damage subsequently propagated to the adjacent plies, leading also to delaminations [31]. Fig. 6b, related to a $t_{H} = 2.4$ mm SS repair, pictures this damage mechanism.
- **Laminate failure**: Failure in the laminate outside the repair region, with no visible damage in the patches. An example of this fracture is shown in Fig. 6c, pertaining to a $L_{O} = 15$ mm DS repair.
- **Partial patch debonding**: Localized cohesive fracture of the adhesive layer at the patch edge near plane B (Fig. 4). Fig. 7a relates to a $t_{H} = 1.8$ mm DS repair, while Fig. 7b presents the respective detail.
- **Patch debonding**: Cohesive failure of the adhesive layer initiating at the patch edge near plane B (Fig. 4) at one of the repair faces, propagating to the entire bond. Fig. 7c represents a $L_{O} = 10$ mm DS repair after debonding of the two patches.

Tables 3 and 4 outline the experimental damage mechanisms ruling $P_{m}$ as a function of $L_{O}$ and $t_{H}$ respectively. In some of the test conditions, designated by the dashed lines, damage occurred in two separate stages. However, $P_{m}$ always corresponded to the first damage stage. The open-hole laminate failed by compression (Fig. 6a). For the majority of the repair geometries the value of $P_{m}$ was related to patch debonding. The $L_{O} = 15$ mm SS and DS repairs and the $t_{H} = 0.6$ mm SS repair are the exception, with the value of $P_{m}$ depending on the laminate strength. For the first ones, this can be explained in light of the larger adhesive shear area and consequent higher flexure capabilities of the patches. The repair before debonding was better than the repairs with smaller values of $L_{O}$ [21,23]. Moreover, as the value of $L_{O}$ increases, the inner region of the bond between the peak shear stresses at the overlap edges progressively becomes lightly stressed due to the differential straining of the laminate and patch along the overlap [3,22], which also helps to the reported modification of the failure mode. For the $t_{H} = 0.6$ mm
SS repair, this behaviour is due to the bigger patch flexibility, leading to smaller magnitude of through-thickness peel and shear peak stresses in the adhesive layer [47]. For the remaining repair geometries, as previously mentioned, failure was related to patch debonding which, compared to these exceptions, can be substanti- ated by the smaller resistant area of the adhesive or smaller allow- able flexibility of the patches owing to their bigger values of \( \tau \), leading to a premature cohesive failure of the adhesive compared to the compressive failure of the laminate.

In the following, an experimental/numerical cooperative study is performed for a few of the configurations tested. It should be firstly noticed that an increasing deviation of stiffness with \( d \) was noticed for all repairs between the experiments and numerical simulations. This was caused by a minor slipping of the repairs in the testing machine grips, leading to an experimental reduction of stiffness during the tests that is not representative of the real behaviour of the composite. The analysis begins with the \( L_0 = 15 \) mm and \( \tau = 2.4 \) mm SS repairs. Fig. 8 corresponds to the numerical patch debonding. For the \( L_0 = 15 \) mm SS repairs (Fig. 8a), the patch debonded after \( r_{\text{c,exp}} \), while for the \( L_0 = 15 \) mm SS repair (Fig. 8b) the opposite scenario occurred. Fig. 9a shows the \( P-d \) curves for the \( L_0 = 15 \) mm SS repairs, with the value of \( P_m \) being related to compression failure (Table 3, Fig. 6b). This causes the minor drop on \( P_m \) observed in the experimental \( P-d \) curves at \( d \approx 0.60 \) mm. The subsequent drop of \( P \) relate to patch debonding and compression failure at plane \( A \) (Fig. 4). Consistently to these results, the abrupt drop of \( P \) on the numerical curve at \( d = 0.80 \) mm was related to patch debonding, after the predicted compression failure and \( P_m \) (the circle marks in the numerical \( P-d \) curves corre- spond to the compression failure prediction). For the \( \tau = 2.4 \) mm SS repairs (Fig. 9b), the experimental drop on \( P_m \) is due to partial patch debonding (Table 4, Fig. 7a and b). A small plateau followed, prior to final failure by complete patch debonding (Fig. 7c) and compression failure of the laminate (Fig. 6b). This behaviour was equally captured by the FEM simulation. In fact, patch debonding (corresponding to the drop on \( P_m \) at \( d \approx 0.55 \) mm) took place prior to \( r_{\text{c,exp}} \). The fracture predictions for the other configurations were also in close agreement with the test results.

The corresponding DS repairs (i.e., \( L_0 = 15 \) mm and \( \tau = 2.4 \) mm) are also addressed in this discussion. Considering the \( L_0 = 15 \) mm DS repair (Fig. 10a), the drop on \( P_m \) of the experimental \( P-d \) curves occurs due to laminate failure outside the repair region (Table 3, Fig. 6c). The numerical approximation shows a value of \( P_m \) of ident- ical magnitude, and compression failure in the laminate occurring after this value. Patch debonding occurs only after \( d \approx 0.82 \) mm, which is consistent with the experiments. However, it should be mentioned that only in this particular case the proposed stress cri- terion is not the most adequate, since the laminate failure occurred mostly by delamination between differently oriented plies (Fig. 6c) instead of compression failure (Fig. 6b). For the \( \tau = 2.4 \) mm DS repair (Fig. 10b), \( P_m \) was ruled by the repair buckling (Fig. 6a). A partial patch debonding event (Table 4, Fig. 7a and b) and a very small plateau region followed, prior to complete failure by patch debond- ing (Fig. 7c) and compression failure (Fig. 6b). This behaviour was obtained numerically, with \( P_m \) related to the onset of global buckling and the abrupt drop at \( d \approx 0.60 \) mm corresponding to par- tial patch debonding followed by compression failure shortly after. The estimated compression failure occurs only after \( P_m \) and patch debonding, which is consistent with the experimental observations.

<table>
<thead>
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<th>( L_0 )</th>
<th>5 mm</th>
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<td>Patch debonding and compression failure</td>
<td>Patch debonding and compression failure</td>
<td>Compression failure</td>
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<tr>
<td>DS repairs</td>
<td>Patch debonding and compression failure</td>
<td>Partial patch debonding</td>
<td>Laminate failure outside the repair region</td>
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</table>

<table>
<thead>
<tr>
<th>( \tau )</th>
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<th>1.2 mm</th>
<th>1.8 mm</th>
<th>2.4 mm</th>
</tr>
</thead>
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<tr>
<td>SS repairs</td>
<td>Compression failure</td>
<td>Patch debonding and compression failure</td>
<td>Patch debonding and compression failure</td>
<td>Partial patch debonding</td>
</tr>
<tr>
<td>DS repairs</td>
<td>Partial patch debonding</td>
<td>Partial patch debonding</td>
<td>Partial patch debonding</td>
<td>Partial patch debonding</td>
</tr>
</tbody>
</table>

*Table 3* Failure modes of the repairs as a function of \( L_0 \).

*Table 4* Failure modes of the repairs as a function of \( \tau \).
5.2. Summary of the results

A comparative analysis is performed between the experimental and numerical results of elastic stiffness \( K \); Fig. 11) and \( P_m \) (Fig. 12) as a function of \( L_o \) and \( t_h \), including the standard deviation of the experiments. \( K \) is defined as the \( P/d \) quotient in the initial part of the \( P-d \) curve. The predictions of \( K \) were accurate as a function of both quantities. The gradual increase of \( K \) with \( L_o \) (Fig. 11a) for both SS and DS repairs is explained in light of the progressive increase of the adhesive shear area and corresponding larger reinforcement capabilities of the patches [21,23]. This also applies to the difference between the SS and DS repairs. On the other hand, \( t_h \) only affects \( K \) up to 1.8 mm for both SS and DS repairs (Fig. 11b). An identical trend between \( P_m \) and \( K \) could be expected under these circumstances, since the value of \( P_m \) for structures under pure compression typically corresponds to the initiation of global buckling, which is determined by their stiffness [60,61]. The characteristic dimensions of structures are also known to greatly influen...
ence $P_{m}$. In fact, Eryigit et al. [29] reported on the buckling loads of thin glass-epoxy laminates with a circular hole at different locations, among which equally to this work, i.e., at the middle of the laminate. For this geometry, the buckling load significantly diminished by increasing the length between grips. A moderate reduction was also found by increasing the hole diameter. Oppositely, the strength of the materials involved typically only influences the failure displacement. However, in this particular situation, the prospect of premature patch debonding and compression failure, prior to global buckling, must be investigated. Actually, some of the tested geometries showed a premature patch debonding before global buckling (Tables 3 and 4), of which the $L_{o} = 2.4$ mm SS repairs are an example (Fig. 9b). On the other hand, the $L_{o} = 15$ mm SS repairs failed by compression prior to global buckling (Fig. 9a). Apart from these exceptions, on all the other repair geometries $P_{m}$ was ruled by the value of $K$, and a plateau on the $P$–$d$ curves was observed prior to the initiation of damage. Based on these assumptions, the tendecies between $K$ and $P_{m}$ showed some similarities. In fact, $P_{m}$ increased with $L_{o}$ for the SS and DS repairs, with an increasing difference between both with $L_{o}$. It should be emphasized that the numerical prediction for the $L_{o} = 15$ mm DS repair corresponds to a criterion (compression failure in the laminate, Fig. 6b) that was not consistent with the experimental failure mode mentioned earlier (laminate failure outside the repair region, Fig. 6c). Fig. 12a also emphasizes on the major difference of $P_{m}$ between the SS and DS repairs. Even though with both repair configurations the strengthening at the notch region increases gradually with $L_{o}$ (thus having in both cases a strength improvement of $P_{m}$ with $L_{o}$), the improvement of $P_{m}$ is much more significant for the DS repairs. This is due to the load eccentricity that exists with the SS repair geometry notwithstanding the value of $L_{o}$, and is pre- vented using DS repairs [3,10]. This eccentric load, which increases with $L_{o}$ due to the corresponding increase of adhesive shear area [42], helps to the lateral flexure of the laminates with correspond- ing smaller strength improvement.

The value of $t_{o}$ showed a bigger influence on $P_{m}$ for the smaller values of $t_{o}$. A significant and similar difference was also found between the SS and DS repairs for all the values of $t_{o}$. Overall, the FEM predictions were accurate. In terms of basic principles for repairing composite structures under buckling compression, for the range of $L_{o}$ and $t_{o}$ values examined, the $L_{o} = 15$ mm DS repairs showed to be the most effective. An acceptable restitution of strength was also attained using DS repairs with $L_{o} = 10$ mm and $1.2 < t_{o} < 2.4$ mm, and also with $L_{o} = 15$ mm SS repairs. However, these particular results shall be considered valid only for the set of material, geometrical and testing conditions selected for this study, since the buckling behaviour of these structures is extremely sensitive to is- sues such as the materials stiffness in the loading direction, load eccentricity and specimen length between grips, amongst other parameters [43]. As a result, when in the presence of different conditions, the repairs should be numerically analysed and optimized prior to their execution.

6. Concluding remarks

An experimental and finite element parametric study was performed on the behaviour of single and double-strap repairs of carbon-epoxy laminates under buckling unrestrained compression. The influence of the overlap length and patch thickness on the failure modes, elastic stiffness and strength of the repairs was evalu- ated, to validate a finite element methodology to simulate adhesively-bonded assemblies. This procedure used a trapezoidal mixed-mode cohesive zone model to simulate the adhesive layer and a stress-based criterion to predict compression failure in the laminate. The cohesive laws of the adhesive layer in pure modes I and II were estimated by an inverse method. The pure mode III cohesive law was equated to the pure mode II one. Results showed that the finite element method can be a valuable predictive tool and an option for the reduction of costs due to experimentation, provided that suitable criteria are employed for the simulation of the different types of fracture. Design principles were proposed to execute single and double-strap repairs on composite structures under compression, which should not be separated from the specific set of conditions selected for the repairs. For the range of overlap length (5–15 mm) and patch thickness values examined (0.6–2.4 mm), the 15 mm overlap length double-strap repairs showed the best results. Good results were also attained using double-strap repairs with 10 mm of overlap length and values of patch thickness between 1.2 and 2.4 mm, and also with single-strap repairs with an overlap length of 15 mm.

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